Ups and Downs

BRITANNICA
Mathematics in Context

Algebra

HOLT, RINEHART AND WINSTON

Mathematics in Context is a comprehensive curriculum for the middle grades. It was developed in 1991 through 1997 in collaboration with the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht, The Netherlands, with the support of the National Science Foundation Grant No. 9054928.

The revision of the curriculum was carried out in 2003 through 2005, with the support of the National Science Foundation Grant No. ESI 0137414.

National Science Foundation

Opinions expressed are those of the authors and not necessarily those of the Foundation.

Abels, M.; de Jong, J. A.; Dekker, T.; Meyer, M. R.; Shew, J. A.; Burrill, G.; and Simon, A. N. (2006). *Ups and downs*. In Wisconsin Center for Education Research & Freudenthal Institute (Eds.), Mathematics in Context. Chicago: Encyclopædia Britannica, Inc.

ISBN 0-03-038576-8

3 4 5 6 073 09 08 07 06

The *Mathematics in Context* Development Team

Development 1991–1997

The initial version of *Ups and Downs* was developed by Mieke Abels and Jan Auke de Jong. It was adapted for use in American schools by Margaret R. Meyer, Julia A. Shew, Gail Burrill, and Aaron N. Simon.

Wisconsin Center for Education Research Staff

Thomas A. Romberg
Director

Joan Daniels Pedro
Assistant to the Director

Gail Burrill
Coordinator

Margaret R. Meyer
Coordinator

Project Staff

Jonathan Brendefur
Laura Brinker
James Browne
Jack Burrill
Rose Byrd
Peter Christiansen
Barbara Clarke
Doug Clarke
Beth R. Cole
Fae Dremock
Mary Ann Fix
Sherian Foster
James A, Middleton
Jasmina Milinkovic
Margaret A. Pligge
Mary C. Shafer
Julia A. Shew
Aaron N. Simon
Marvin Smith
Stephanie Z. Smith
Mary S. Spence

Freudenthal Institute Staff

Jan de Lange
Director

Els Feijs
Coordinator

Martin van Reeuwijk
Coordinator

Project Staff

Mieke Abels
Nina Boswinkel
Frans van Galen
Koeno Gravemeijer
Marja van den Heuvel-Panhuizen
Jan Auke de Jong
Vincent Jonker
Ronald Keijzer
Martin Kindt
Jansie Niehaus
Nanda Querelle
Anton Roodhardt
Leen Streefland
Adri Treffers
Monica Wijers
Astrid de Wild

Revision 2003–2005

The revised version of *Ups and Downs* was developed by Truus Dekker and Mieke Abels. It was adapted for use in American schools by Gail Burrill.

Wisconsin Center for Education Research Staff

Thomas A. Romberg
Director

David C. Webb
Coordinator

Gail Burrill
Editorial Coordinator

Margaret A. Pligge
Editorial Coordinator

Project Staff

Sarah Ailts
Beth R. Cole
Erin Hazlett
Teri Hedges
Karen Hoiberg
Carrie Johnson
Jean Krusi
Elaine McGrath
Margaret R. Meyer
Anne Park
Bryna Rappaport
Kathleen A. Steele
Ana C. Stephens
Candace Ulmer
Jill Vettrus

Freudenthal Institute Staff

Jan de Lange
Director

Truus Dekker
Coordinator

Mieke Abels
Content Coordinator

Monica Wijers
Content Coordinator

Project Staff

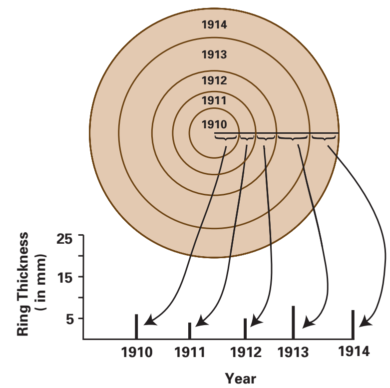
Arthur Bakker
Peter Boon
Els Feijs
Dédé de Haan
Martin Kindt
Nathalie Kuijpers
Huub Nilwik
Sonia Palha
Nanda Querelle
Martin van Reeuwijk

Cover photo credits: (left to right) © William Whitehurst/Corbis; © Getty Images; © Comstock Images

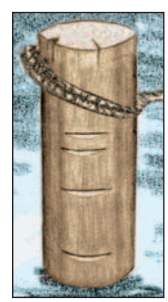
Illustrations
1, 13 Holly Cooper-Olds; **18, 19** (bottom), **22** Megan Abrams/ © Encyclopædia Britannica, Inc.; **35** Holly Cooper-Olds

Photographs
4 Sam Dudgeon/HRW; **7** (middle left) © PhotoDisc/Getty Images; (middle right) © Jack Hollingsworth/PhotoDisc/Getty Images; (bottom) © Corbis; **8** © ImageState; **14** © Stephanie Pilick/AFP/Getty Images; **17** Stephanie Friedman/HRW; **27** John Bortniak, NOAA; **29** © Kenneth Mantai/Visuals Unlimited; **31** (top) Peter Van Steen/HRW Photo; (bottom) CDC; **37** © Corbis; **38** Dynamic Graphics Group/Creatas/Alamy; **39** © Corbis; **43** Victoria Smith/HRW; **44** Sam Dudgeon/HRW Photo; **48** (left to right) © Corbis; © Digital Vision/Getty Images

Contents



1914
1913
1912
1911
1910
Ring Thickness (in mm)
25
15
5
1910
1911
1912
1913
1914
Year



Dear Student,

Welcome to *Ups and Downs*. In this unit, you will look at situations that change over time, such as blood pressure or the tides of an ocean. You will learn to represent these changes using tables, graphs, and formulas.

Graphs of temperatures and tides show up-and-down movement, but some graphs, such as graphs for tree growth or melting ice, show only upward or only downward movement.

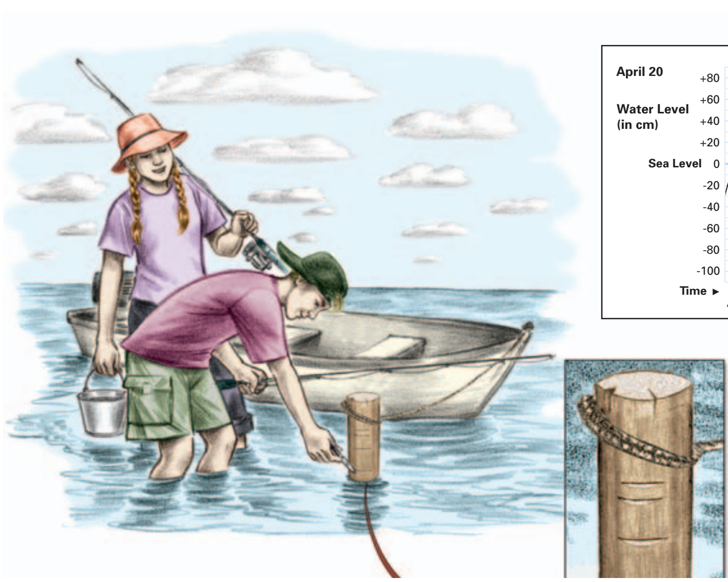
As you become more familiar with graphs and the changes that they represent, you will begin to notice and understand graphs in newspapers, magazines, and advertisements.

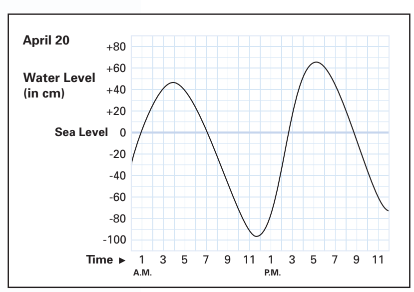
During the next few weeks, look for graphs and statements about growth, such as "Fast-growing waterweeds in lakes become a problem." Bring to class interesting graphs and newspaper articles and discuss them.

Telling a story with a graph can help you understand the story.

Sincerely,

The Mathematics in Context Development Team





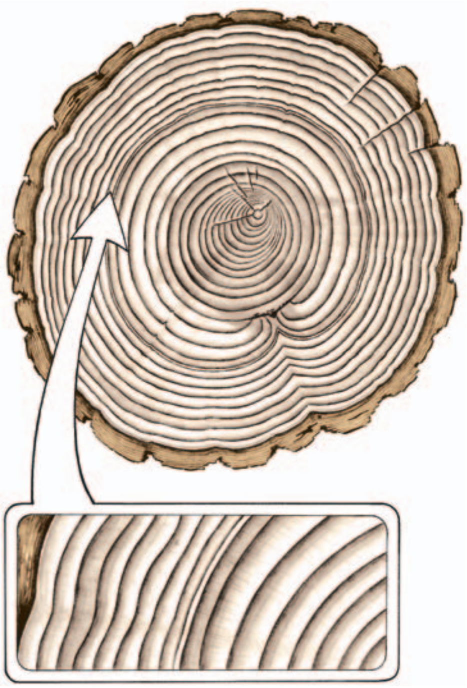
A Trendy Graphs

Wooden Graphs

Giant sequoia trees grow in Sequoia National Park in California. The largest tree in the park is thought to be between 3,000 and 4,000 years old.

It takes 16 children holding hands to reach around the giant sequoia shown here.

1. Find a way to estimate the circumference and diameter of this tree.



This is a drawing of a cross section of a tree. Notice its distinct ring pattern. The bark is the dark part on the outside. During each year of growth, a new layer of cells is added to the older wood. Each layer forms a ring. The distance between the dark rings shows how much the tree grew that year.

2. Look at the cross section of the tree. Estimate the age of this tree. How did you find your answer?

Take a closer look at the cross section. The picture below the cross section shows a magnified portion.

3. a. Looking at the magnified portion, how can you tell that this tree did not grow the same amount each year?

b. **Reflect** What are some possible reasons for the tree's uneven growth?

Tree growth is directly related to the amount of moisture supplied. Look at the cross section on page 1 again. Notice that one of the rings is very narrow.

4. **a.** What conclusion can you draw about the rainfall during the year that produced the narrow ring?

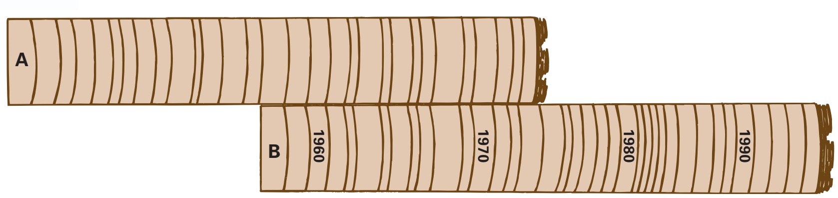
b. How old was the tree that year?

The oldest known living tree is a bristlecone pine (*Pinus aristata*) named Methuselah. Methuselah is about 4,700 years old and grows in the White Mountains of California.

It isn't necessary to cut down a tree in order to examine the pattern of rings. Scientists use a technique called **coring** to take a look at the rings of a living tree. They use a special drill to remove a piece of wood from the center of the tree. This piece of wood is about the thickness of a drinking straw and is called a *core sample*. The growth rings show up as lines on the core sample.

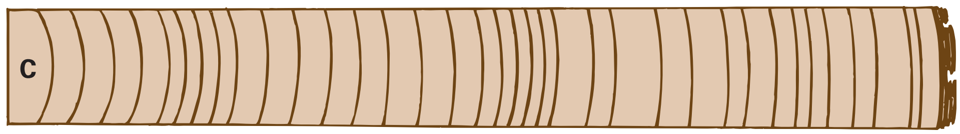
By matching the ring patterns from a living tree with those of ancient trees, scientists can create a calendar of tree growth in a certain area.

The picture below shows how two core samples are matched up. Core sample B is from a living tree. Core sample A is from a tree that was cut down in the same area. Matching the two samples in this way produces a "calendar" of wood.



5. In what year was the tree represented by core sample A cut down?

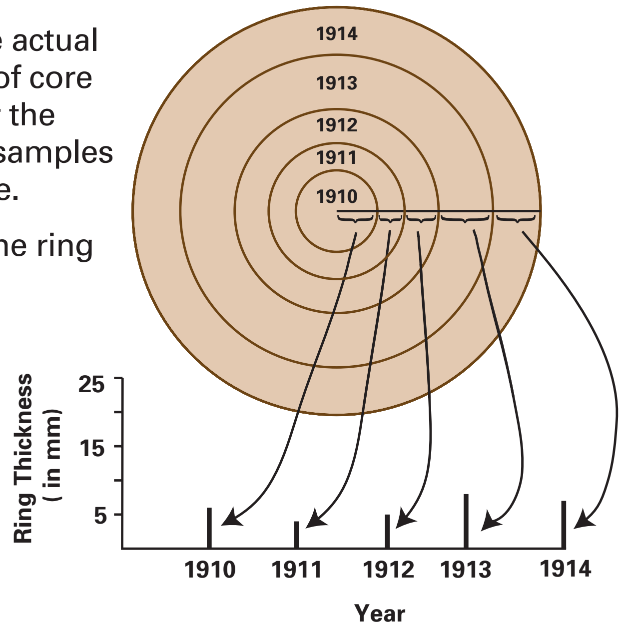
The next picture shows a core sample from another tree that was cut down. If you match this one to the other samples, the calendar becomes even longer. Enlarged versions of the three strips can be found on **Student Activity Sheet 1**.



6. What period of time is represented by the three core samples?

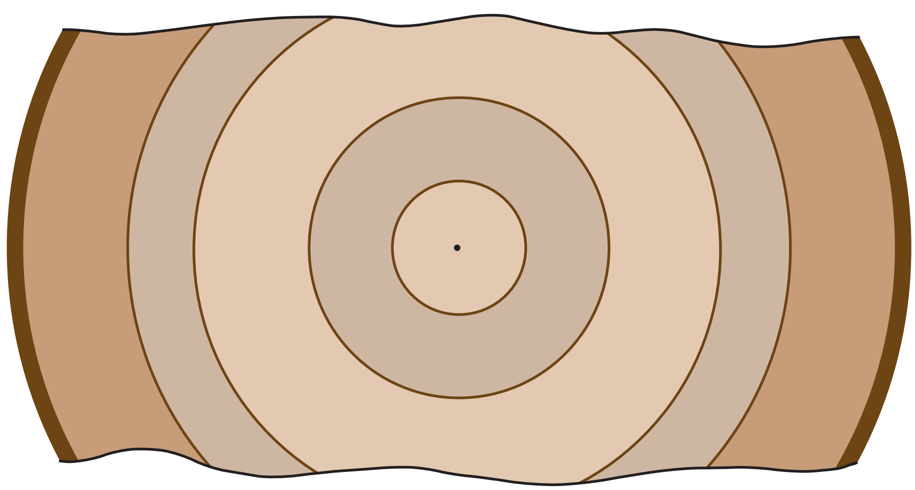
Instead of working with the actual core samples or drawings of core samples, scientists transfer the information from the core samples onto a diagram like this one.

7. About how thick was the ring in 1910?

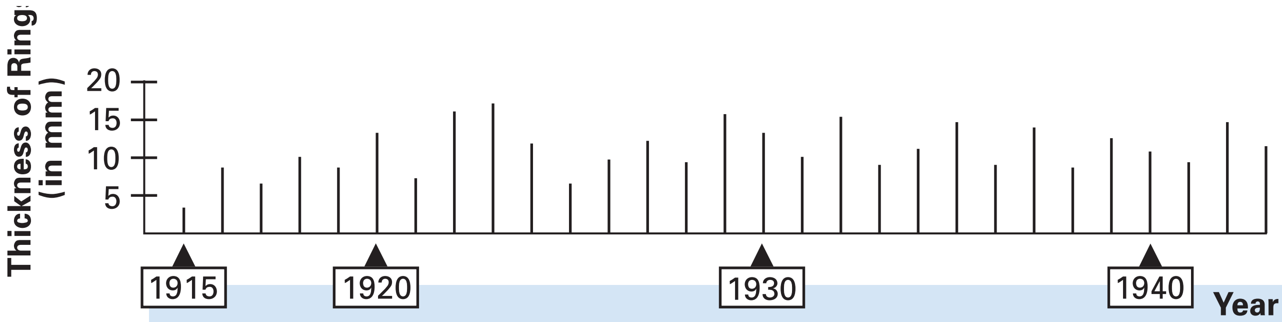


Totem Pole

Tracy found a totem pole in the woods behind her house. It had fallen over, so Tracy could see the growth rings on the bottom of the pole. She wondered when the tree from which it was made was cut down.



Tracy asked her friend Luis, who studies plants and trees in college, if he could help her find the age of the wood. He gave her the diagram pictured below, which shows how cedar trees that were used to make totem poles grew in their area.



8. **a.** Make a similar diagram of the thickness of the rings of the totem pole that Tracy found.

b. Using the diagram above, can you find the age of the totem pole? What year was the tree cut down?

Growing Up

On Marsha's birthday, her father marked her height on her bedroom door. He did this every year from her first birthday until she was 19 years old.

9. There are only 16 marks. Can you explain this?

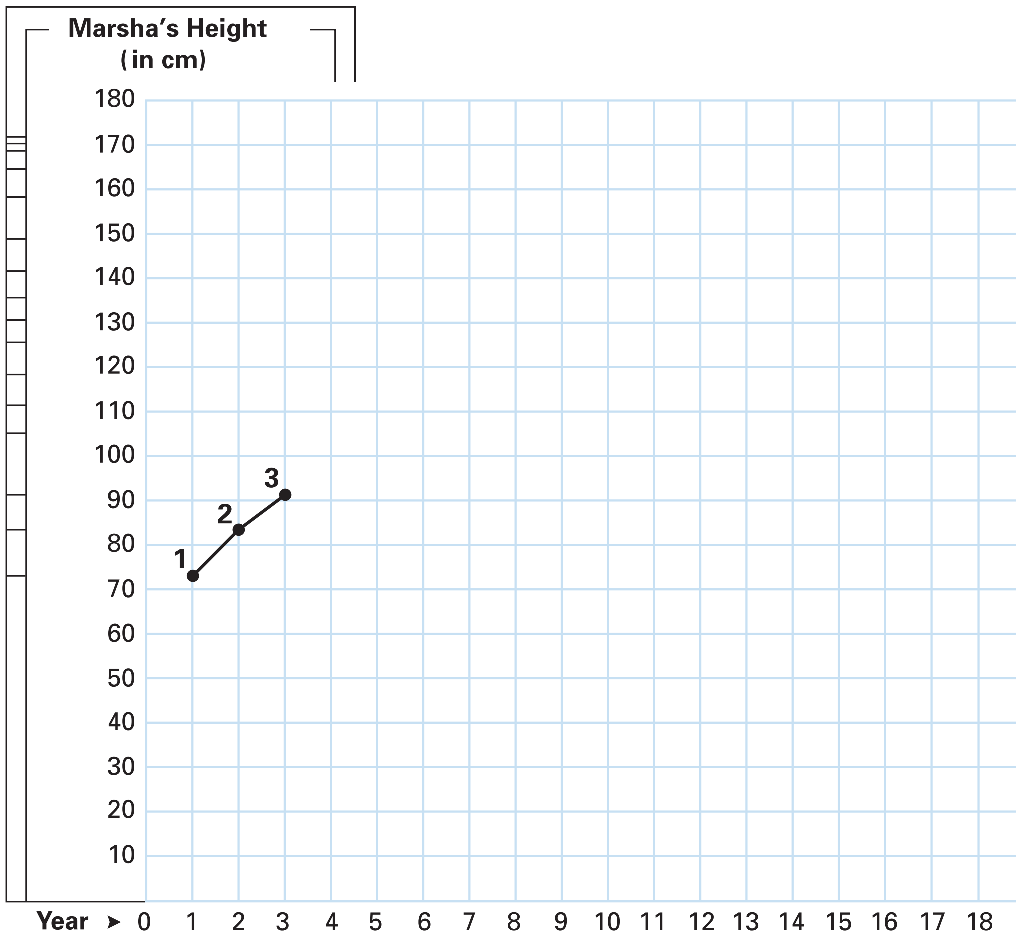
10. How old was Marsha when her growth slowed considerably?

11. Where would you put a mark to show Marsha's height at birth?

12. **a.** Use **Student Activity Sheet 2** to draw a graph of Marsha's growth. Use the marks on the door to get the vertical coordinates. Marsha's height was 52 centimeters (cm) at birth.

    b. How does the graph show that Marsha's growth slowed down at a certain age?

    c. How does the graph show the year during which she had her biggest growth spurt?



The graph you made for problem 12 is called a **line graph** or plot over time. It represents information occurring over time. If you connect the ends of the segments of the graph you made for problem 8 on page 4, you would also see a plot over time of the differences in growth of the tree from one year to the next. These graphs show a certain trend, like how Marsha has grown or how the tree grew each year. You cannot write one formula or equation to describe the growth.

Growth Charts

Healthcare workers use growth charts to help monitor the growth of children up to age three.

13. Why is it important to monitor a child's growth?

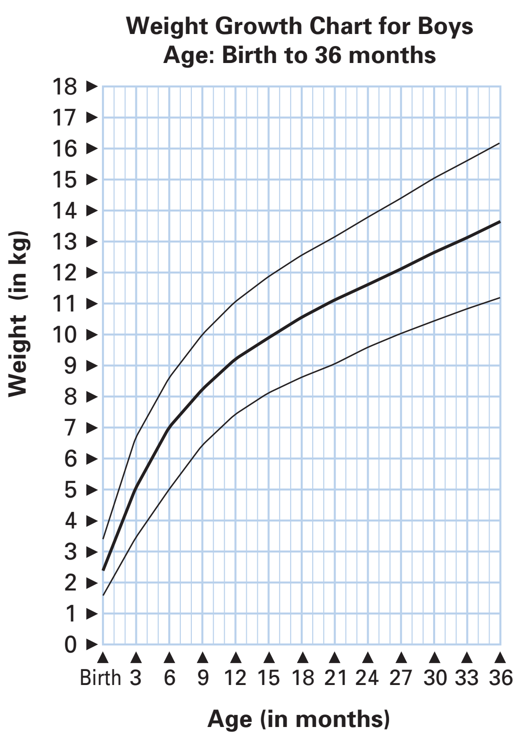
The growth chart below shows the weight records, in kilograms (kg), of a 28-month-old boy.

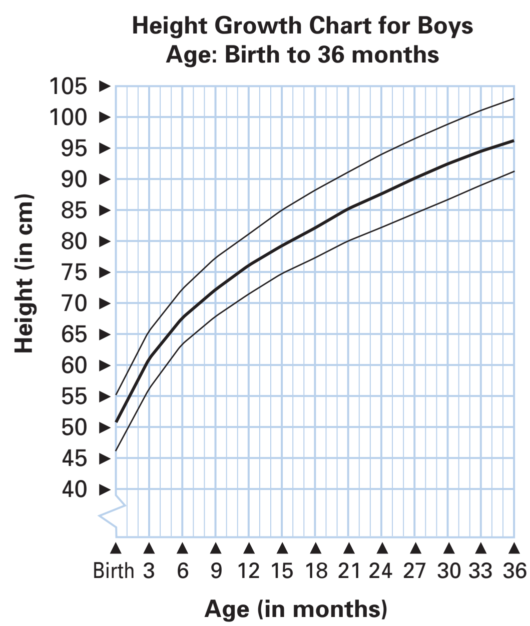
Month	Birth	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weight	2.7	3.6	5.7	7.0	7.3	7.8	8.0	8.8	8.8	8.8	9.3	9.6	10.5	10.3	11.3	12.0
Month	16	17	18	19	20	21	22	23	24	25	26	27	28			
Weight	12.4	12.9	13.1	12.9	10.5	9.2	9.5	12.0	13.0	13.6	13.5	14.0	14.2			

14. What conclusion can you draw from this table? Do you think this boy gained weight in a "normal" way?

The graphs that follow show normal ranges for the weights and heights of young children in one country. The normal growth range is indicated by curved lines.

Note: The zigzag line on the lower left of the height graph indicates that the lower part of the graph, from 0–40, is omitted.





15. Describe how the growth of a "normal" boy changes from birth until the age of three.

In both graphs, one curved line is thicker than the other two.

16. a. What do these thicker curves indicate?

b. These charts are for boys. How do you think charts for girls would differ from these?

17. a. Graph the weight records from problem 13 on the weight growth chart on **Student Activity Sheet 3**.

b. Study the graph that you made. What conclusions can you draw from the graph?

Here are four weekly weight records for two children. The records began when the children were one year old.

	Week 1	Week 2	Week 3	Week 4
Samantha's Weight (in kg)	11.8	11.6	11.3	10.9
Hillary's Weight (in kg)	10.5	10.0	9.7	9.5

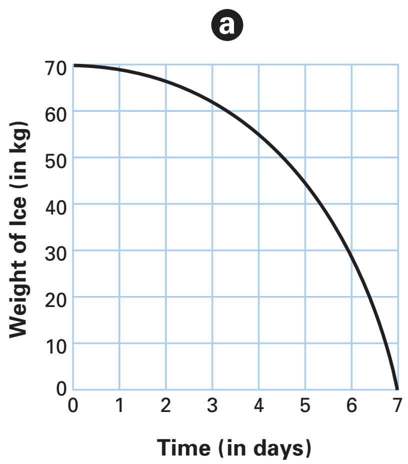
18. Although both children are losing weight, which one would you worry about more? Why?

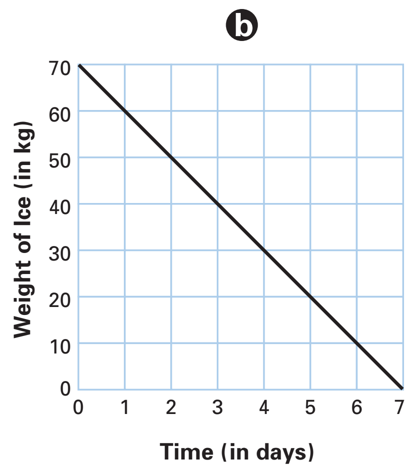
Water for the Desert

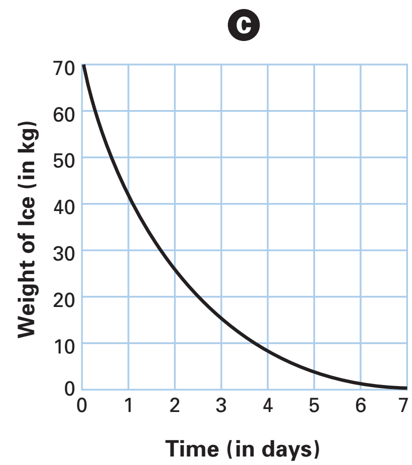
In many parts of the world, you can find deserts near the sea. Because there is a water shortage in the desert, you might think that you could use the nearby sea as a water source. Unfortunately, seawater contains salt that would kill the desert plants.

Some scientists are investigating ways to bring ice from the Antarctic Ocean to the desert. The ice from an iceberg is made from fresh water. It is well packed and can be easily pulled by boat. However, there is one problem: The ice would melt during the trip, and the water from the melted ice would be lost.

There are different opinions about how the iceberg might melt during a trip. The three graphs illustrate different opinions.







The graphs are not based on data, but they show possible trends.

19. **Reflect** Use the graphs to describe, in your own words, what the three opinions are.

Sunflowers

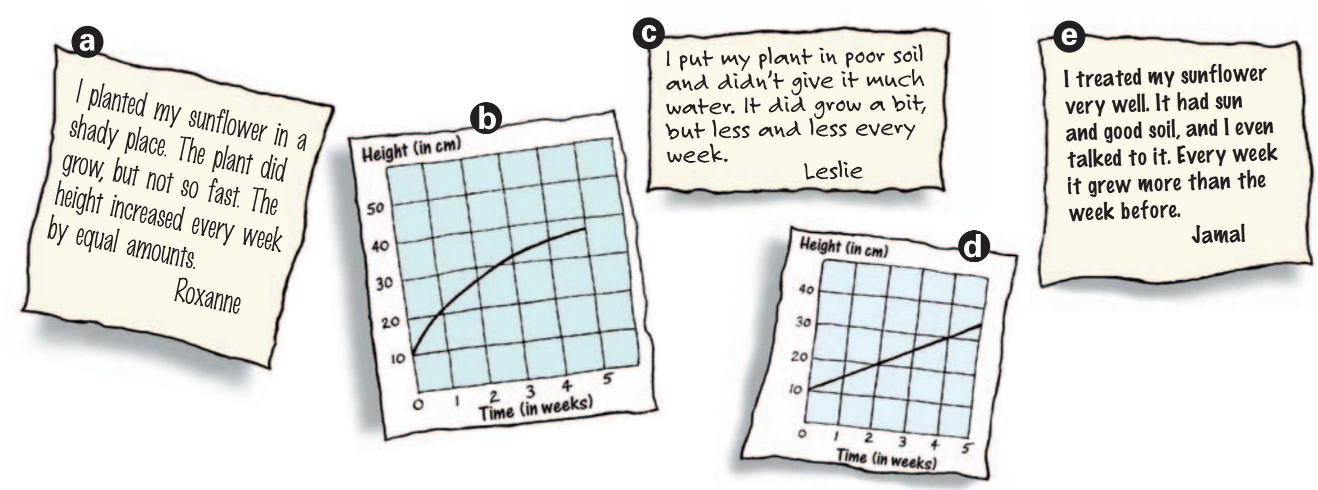
Roxanne, Jamal, and Leslie did a group project on sunflower growth for their biology class. They investigated how different growing conditions affect plant growth. Each student chose a different growing condition.

The students collected data every week for five weeks. At the end of the five weeks, they were supposed to write a group report that would include a graph and a story for each of three growing conditions.

Unfortunately, when the students put their work together, the pages were scattered, and some were lost. The graphs and written reports that were left are shown on the next page.

20. **a.** Find which graph and written report belong to each student.

b. Create the missing graph.



The type of growth displayed by Roxanne's sunflower is called **linear growth**.

21. Why do you think it is called linear growth?

A plant will hardly ever grow in a linear way all the time, but for some period, the growth might be linear. Consider a sunflower that has a height of 20 cm when you start your observation and grows 1.5 cm per day.

22. a. In your notebook, copy and fill in the table.

Time (in days)	0	1	2	3	4	5	6	7	8
Height (in cm)	20	21.5							

b. Meryem thinks this is a ratio table. Is she right? Explain your answer.

c. How does the table show linear growth?

d. Use your table to draw a graph. Use the vertical axis for height (in centimeters) and the horizontal axis for time (in days). Label the axes.

Here is a table with data from another sunflower growth experiment.

Time (in weeks)	0	1	2	3	4	5	6
Height (in cm)	10	12.5	17.5	25	35	47.5	

23. a. How can you be sure that the growth during this period was not linear?

b. In your own words, describe the growth of this plant.

A Trendy Graphs

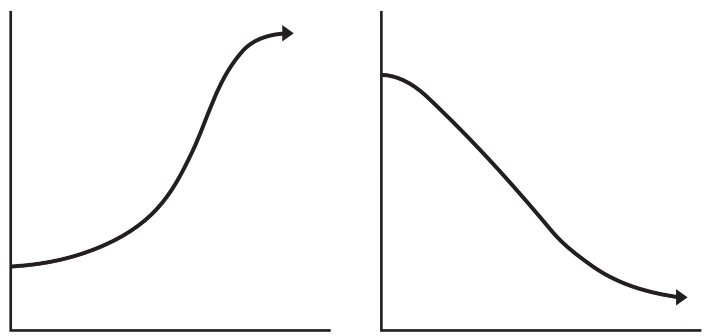
Summary

Information about growth over time can be obtained by looking at:

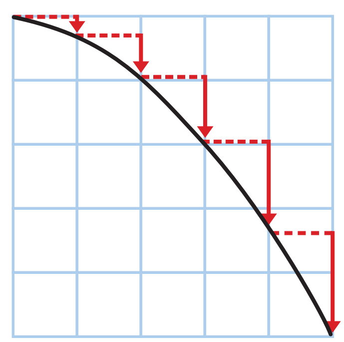
- a statement like "My sunflower grew faster and faster."
- growth "calendars" like the tree rings appearing on a core sample or the height marks made on a door.
- tables like the growth charts used for babies.
- line graphs. The line graphs used in this section show trends.

You can draw a graph by using the information in a table. Often, the graph will give more information than the table.

By looking at a graph, you can see whether and how something is increasing or decreasing over time.



The shape of a graph shows how a value increases or decreases. The following graph and table show a value that is decreasing more and more.

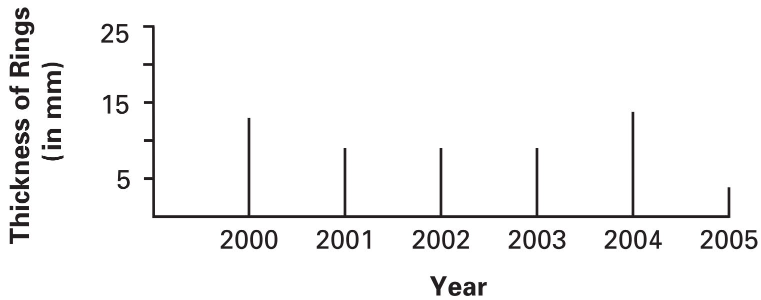


Days	0	1	2	3
Weight	25	23	19	10

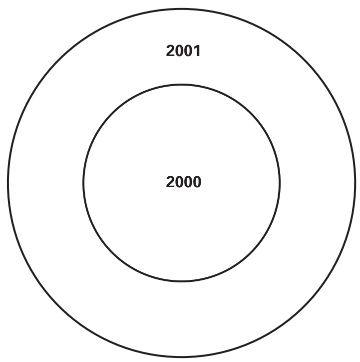
−2 −4 −9

Check Your Work

This diagram represents the thickness of annual rings of a tree. It shows how much the tree grew each year.



1. **a.** Use a ruler and a compass to draw a cross section of this tree. The first two rings are shown here. Copy and continue this drawing to show the complete cross section.



b. Write a story that describes how the tree grew.

This table shows Dean's growth.

Age (in years)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Height (in cm)	80	95	103	109	114	118	124	130	138	144	150	156	161	170	176	181	185	187

2. **a.** Draw a line graph of Dean's height on **Student Activity Sheet 4.**

b. What does this graph show that is not easy to see in the table?

c. At what age did Dean have his biggest growth spurt?

Mr. Akimo owns a tree nursery. He measures the circumference of the tree trunks to check their growth. One spring, he selected two trees of different species to study. Both had trunks that measured 2 inches in circumference. For the next two springs, he measured the circumference of both tree trunks. The results are shown in the table.

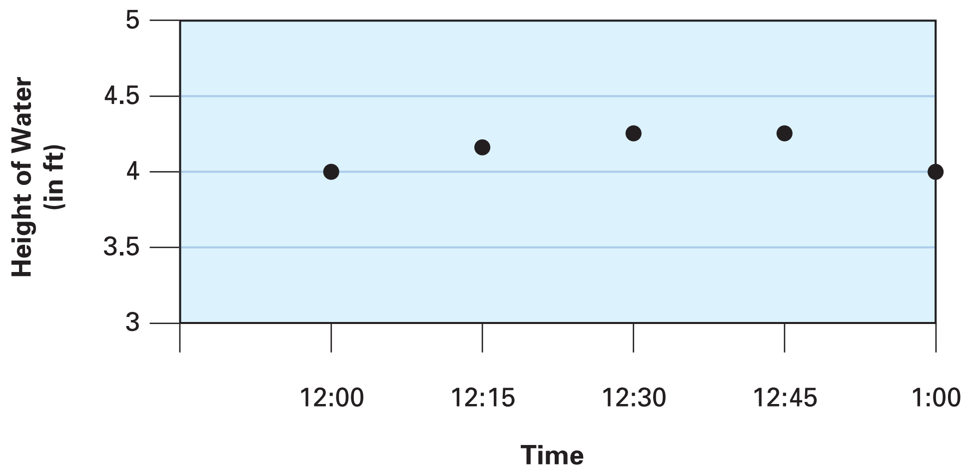
	Circumference (in inches)		
	First Measurement	Second Measurement	Third Measurement
Tree 1	2.0	3.0	4.9
Tree 2	2.0	5.5	7.1

3. a. Which tree will most likely have the larger circumference when Mr. Akimo measures them again next spring? Explain how you got your answer.

b. **Reflect** Do you get better information about the growth of the circumference of these trees by looking at the tables or by looking at the graphs? Explain your answer.

For Further Reflection

This graph indicates the height of water in a swimming pool from 12:00 noon to 1:00 P.M. Write a story that describes why the water levels change and at what times. Be specific.



B Linear Patterns

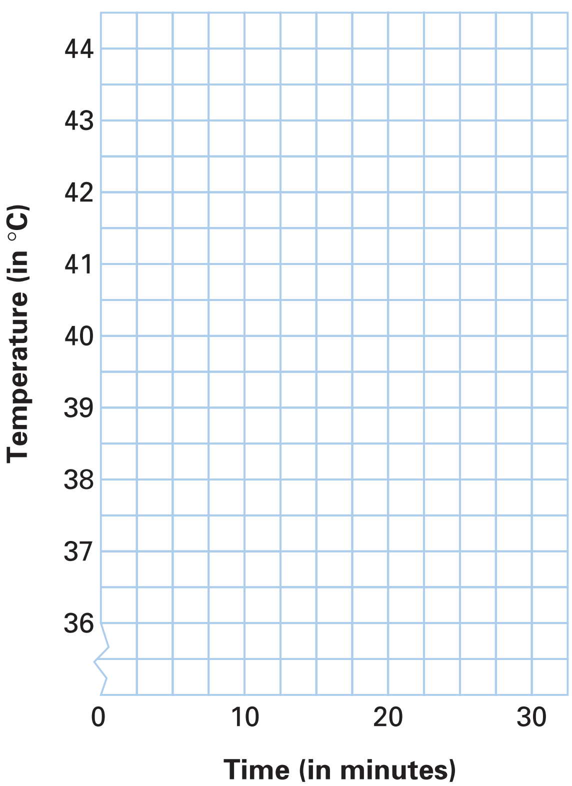
The Marathon

In 490 B.C., there was a battle between the Greeks and the Persians near the village of Marathon. Legend tells us that immediately after the Greeks won, a Greek soldier was sent from Marathon to Athens to tell the city the good news. He ran the entire 40 kilometers (km). When he arrived, he was barely able to stammer out the news before he died.

1. What might have caused the soldier's death?

Marathon runners need lots of energy to run long distances. Your body gets energy to run by burning food. Just like in the engine of a car, burning fuel generates heat. Your body must release some of this heat or it will be seriously injured.

Naoko Takahashi won the women's marathon during the 2000 Olympics. She finished the race in 2 hours, 23 minutes, and 14 seconds. She was the first Japanese woman to win an Olympic gold medal in track and field. Today the marathon is 42.195 km long, not the original 40.



Normal body temperature for humans is 37° Centigrade (C), or 98.6° Fahrenheit (F). At a temperature of 41°C (105.8°F), the body's cells stop growing. At temperatures above 42°C (107.6°F), the brain, kidneys, and other organs suffer permanent damage.

When you run a marathon, your body produces enough heat to cause an increase in body temperature of 0.17°C every minute.

2. **a.** Make a table showing how your body temperature would rise while running a marathon if you did nothing to cool off. Show temperatures every 10 minutes.

b. Use the table to make a graph of this data on **Student Activity Sheet 5.**

3. **a.** Why is the graph for problem 2b not realistic?

b. What does your body do to compensate for the rising temperature?

When the body temperatures of marathon runners rise by about 1°C, their bodies begin to sweat to prevent the temperature from rising further. Then the body temperature neither increases nor decreases.

4. Use this information to redraw the line graph from problem 2b on **Student Activity Sheet 5.**

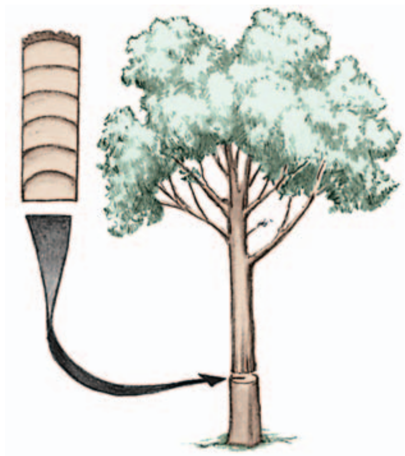
During the race, the body will lose about $\frac{1}{5}$ of a liter of water every 10 minutes.

5. How much water do you think Naoko Takahashi lost during the women's marathon in the 2000 Olympics?

What's Next?

Here you see a core sample of a tree. When this sample was taken, the tree was six years old.

6. What can you tell about the growth of the tree?



The table shows the radius for each year. Remember that the radius of a circle—in this problem, a growth ring—is half the diameter.

Year	1	2	3	4	5	6
Radius (in mm)	4	8	12	16	20	24

+4 +4 +4 +4 +4

7. Use the table to draw a graph.

The graph you made is a straight line. Whenever a graph is a straight line, the growth is called linear growth. (In this case, the tree grew linearly.) In the table, you can see the growth is linear because the differences in the second row are equal. The change from one year to the next was the same for all of the years.

8. **a.** What might have been the size of the radius in year 7? Explain how you found your answer.

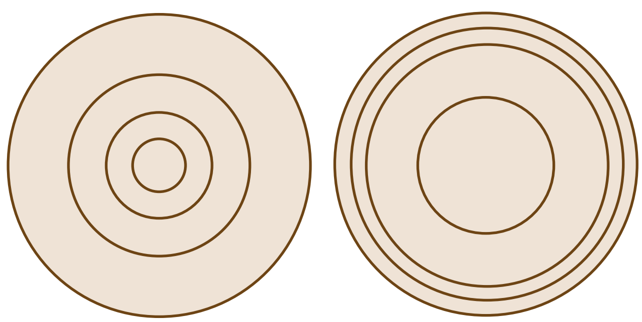
b. Suppose the tree kept growing in this way. One year the radius would be 44 millimeters (mm). What would the radius be one year later?

If you know the radius of the tree in a certain year, you can always find the radius of the tree in the year that follows if it keeps growing linearly. In other words, if you know the radius of the CURRENT year, you can find the radius of the NEXT year.

9. If this tree continues growing linearly, how can you find the radius of the NEXT year from the radius of any CURRENT year? Write a formula.

The formula you wrote in problem 9 is called a **NEXT-CURRENT formula**.

Here you see the cross sections of two more trees. You could make graphs showing the yearly radius for each of these trees, too.



10. **a.** Will the graphs be straight lines or not? How can you tell without drawing the graphs?

b. Describe the shape of the graph for each tree. You may want to make a graph first.

Hair and Nails

Paul went to get a haircut. When he got home, he looked in the mirror and screamed, "It's too short!"

He decided not to get his hair cut again for a long time. In the meantime, he decided to measure how fast his hair grew. Below is a table that shows the length of Paul's hair (in centimeters) as he measured it each month.

Time (in months)	0	1	2	3	4	5	6
Length (in cm)	2	3.5	5	6.5			

11. How long was Paul's hair after the haircut?

12. a. How long will his hair be in five months?

b. Why is it easy to calculate this length?

13. a. How long will Paul's hair be after a year if it keeps growing at the same rate and he does not get a haircut?

b. Draw a graph showing how Paul's hair grows over a year if he does not get a haircut.

c. Describe the shape of this graph.

14. If Paul's hair is 10 cm long at some point, how long will it be one month later?

If you know the length of Paul's hair in the current month, you can use it to find his hair length for the next month.

15. Write a formula using *NEXT* and *CURRENT*.

You knew that the beginning length of Paul's hair was 2 cm. That's why it is possible to make a **direct formula** for Paul's hair growth:

$$L = 2 + 1.5T$$

16. a. What do you think the letter L stands for? The letter T?

b. Explain the numbers in the formula.

Sacha's hair is 20 cm long and grows at a constant rate of 1.4 cm a month.

17. Write a direct formula with *L* and *T* to describe the growth of Sacha's hair.

Time (in months)	Fingernail Length (in mm)
0	15
1	
2	
3	
4	23

Suppose you decided not to cut one fingernail for several months, and the nail grew at a constant rate. The table shows the lengths of a nail in millimeters at the beginning and after four months.

18. How much did this nail grow every month?

19. Predict what the graph that fits the data in the table looks like. If you cannot predict its shape, think of some points you might use to draw the graph.

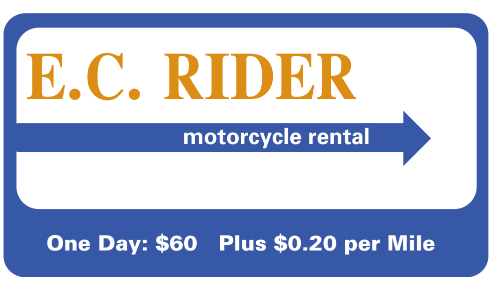
20. Write a direct formula for fingernail growth using *L* for length (in millimeters) and *T* for time (in months).

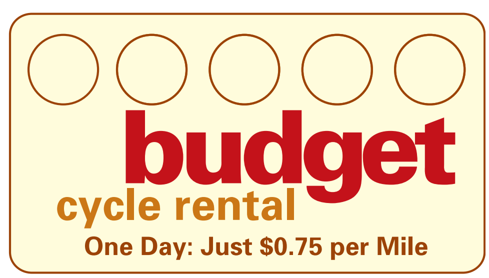
Renting a Motorcycle

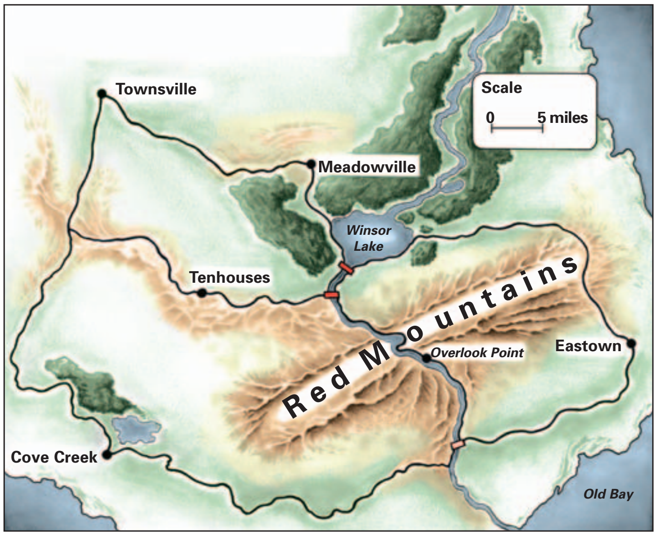
During the summer months, many people visit Townsville. A popular tourist activity there is to rent a motorcycle and take a one-day tour through the mountains.

You can rent motorcycles at E.C. Rider Motorcycle Rental and at Budget Cycle Rental. The two companies calculate their rental prices in different ways.

The most popular trip this season goes from Townsville, through Cove Creek, to Overlook Point, and back through Meadowville.







Even though more and more people are making this 170-mile trip, the owner of Budget Cycle Rental noticed that her business is getting worse. This is very surprising to her, because her motorcycles are of very good quality.

21. **Reflect** What do you think explains the decrease in Budget's business compared to E.C. Rider's?

The rental price you pay depends on the number of miles you ride. With Budget Cycle Rental, the price goes up $0.75 for every mile you ride.

22. **a.** How much does the cost go up per mile with a rental from E.C. Rider?

b. Does that mean it is always less expensive to rent from E.C. Rider? Explain your answer.

Budget Cycle Rental uses this rental formula: $P = 0.75M$.

23. **a.** Explain each part of this formula.

b. What formula does E.C. Rider use?

c. Graph both formulas on **Student Activity Sheet 6**.

Ms. Rider is thinking about changing the rental price for her motorcycles. This will also change her formula. She thinks about raising the starting amount from $60 to $70.

24. **a.** What would the new formula be?

b. Do you think Ms. Rider's idea is a good one? Why or why not?

Budget Cycle Rental is going to change prices too. See the new sign.

25. **a.** Write the new formula for Budget Cycle Rental.

b. Make a graph of this new formula on **Student Activity Sheet 6.** You may want to make a table first.

26. Look again at the 170-mile trip from Townsville. Whom would you rent your motorcycle from now, given the new information from problems 24 and 25?

B Linear Patterns

Summary

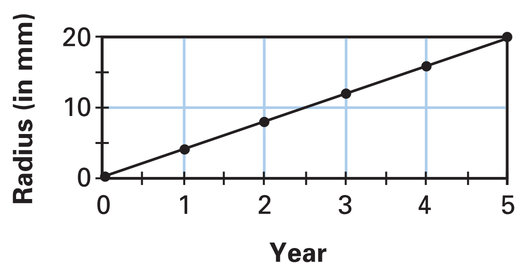
The situations in this section were all examples of graphs with straight lines. A graph with a straight line describes *linear growth*. The rate of change is constant. The differences over *equal time periods* will always be the same.

You can recognize linear growth by looking at the differences in a table or by considering the shape of the graph.

Year	1	2	3	4	5
Radius (in mm)	4	8	12	16	20

+4 +4 +4 +4

Core Sample of a Tree



Linear growth can be described using formulas. A *NEXT-CURRENT formula* that fits this table and graph is:

$$\text{NEXT} = \text{CURRENT} + 4$$

A *direct formula* that fits this table and graph is:

$$\text{radius} = \text{year number} \times 4 \quad \text{or} \quad R = 4Y$$

with the radius measured in millimeters.

Check Your Work

1. Lucia earns $12 per week babysitting.

   a. Make a table to show how much money Lucia earns over six weeks.

   b. Write a formula using *NEXT* and *CURRENT* to describe Lucia's earnings.

   c. Write a direct formula using *W* (week) and *E* (earnings) to describe Lucia's earnings.

Time (in weeks)	0	1	2	3
Length (in cm)	11	12.4	13.8	15.2

2. a. Show that the growth described in the table is linear.

   b. Write a formula using *NEXT* and *CURRENT* for the example.

   c. Write a direct formula using *L* (length) and *T* (time) for the example.

Sonya's hair grew about 14.4 cm in one year. It is possible to write the following formulas:

$$\text{NEXT} = \text{CURRENT} + 14.4$$

$$\text{NEXT} = \text{CURRENT} + 1.2$$

3. Explain what each formula represents.

B Linear Patterns

Lamar has started his own company that provides help for people who have problems with their computer. On his website, he uses a sign that reads:

> **HELP needed for computer problems? We visit you at your home. You pay only $12.00 for the house call and $10.00 for each half hour of service!**

4. Write a direct formula that can be used by Lamar's company.

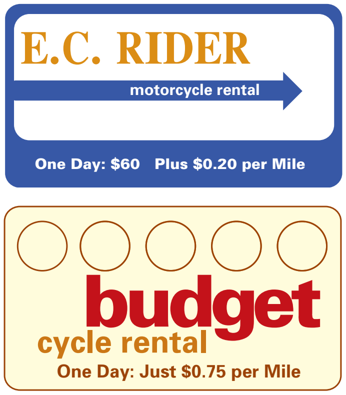
Suppose you want to start your own help desk for computer problems. You want to be less costly than Lamar, and you suppose that most jobs will not take over two hours.

5. **a.** Make your own sign for a website.

b. Make a direct formula you can use. Show why your company is a better choice than Lamar's.

For Further Reflection

Refer to the original prices for E.C. Rider Motorcyle Rentals and Budget Rental Cycles. Describe in detail a trip that would make it better to rent from Budget than from E.C. Rider.



Differences in Growth

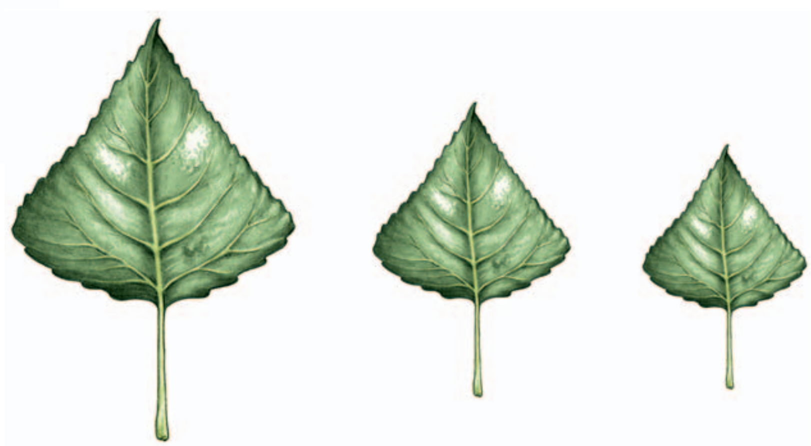
Leaf Area

The main function of leaves is to create food for the entire plant. Each leaf absorbs light energy and uses it to decompose the water in the leaf into its elements—hydrogen and oxygen. The oxygen is released into the atmosphere. The hydrogen is combined with carbon dioxide from the atmosphere to create sugars that feed the plant. This process is called *photosynthesis*.

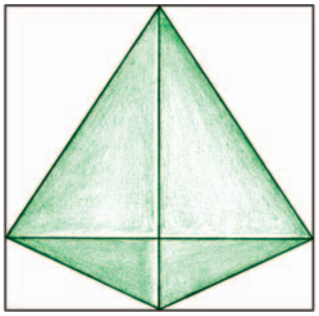
1. **a.** Why do you think a leaf's ability to manufacture plant food might depend on its surface area?

b. Describe a way to find the surface area of any of the leaves shown on the left.

The picture below shows three poplar leaves. Marsha states, "These leaves are similar. Each leaf is a reduction of the previous one."

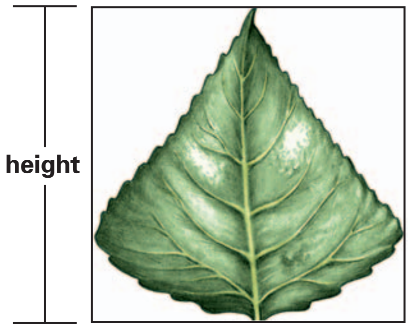


2. Measure the height and width of each of the leaves to determine whether Marsha is right.



One way to estimate the surface area of a poplar leaf is to draw a square around it as shown in the diagram on the right.

The kite-shaped model on the left covers about the same portion of the square as the actual leaf on the left.



3. a. Approximately what portion of the square does the leaf cover? Explain your reasoning.

   b. If you know the height (h) of such a leaf, write a direct formula that you can use to calculate its area (A).

   c. If h is measured in centimeters, what units should be used to express A?

   d. The formula that you created in part **b** finds the area of poplar leaves that are symmetrical. Draw a picture of a leaf that is not symmetrical for which the formula will still work.

Area Differences

You can use this formula for the area of a poplar leaf when the height (h) is known:

$$A = \frac{1}{2}h^2$$

You can rewrite the formula using arrow language:

$$h \xrightarrow{\text{squared}} \;.....\; \xrightarrow{\times \frac{1}{2}} A$$

The table shows the areas of two poplar leaves.

Height (in cm)	6	7	8	9	10	11	12
Area (in cm^2)	18	24.5					

4. a. Verify that the areas for heights of 6 cm and 7 cm are correct in the table.

   b. On **Student Activity Sheet 7**, fill in the remaining area values in the table. Describe any patterns that you see.

   c. **Reflect** How do you know that the relationship between area and height is not linear?

The diagram below shows the differences between the areas of the first three leaves in the table.

Height (in cm)	6	7	8	9	10	11	12
Area (in cm^2)	18	24.5	32	?	?	?	?

First Difference 6.5 7.5 ? ? ? ?

5. a. On **Student Activity Sheet 7**, fill in the remaining "first difference" values. Do you see any patterns in the differences?

   b. The first "first difference" value (6.5) is plotted on the graph on **Student Activity Sheet 8.** Plot the rest of the differences that you found in part **a** on this graph.

   c. Describe your graph.

Height (in cm)	6	7	8	9	10	11	12
Area (in cm^2)	18	24.5	32	?	?	?	?

First Difference 6.5 7.5 ? ? ? ?

Second Difference 1 1 ? ? ?

As shown in the diagram, you can find one more row of differences, called the **second differences**.

6. a. Finish filling in the row of second differences in the diagram on **Student Activity Sheet 7**.

   b. What do you notice about the second differences? If the diagram were continued to the right, find the next two second differences.

   c. How can you use the patterns of the second differences and first differences to find the areas of leaves that have heights of 13 cm and 14 cm? Continue the diagram on **Student Activity Sheet 7** for these new values.

   d. Use the area formula for poplar leaves ($A = \frac{1}{2}h^2$) to verify your work in part **c.**

7. **a.** What is the value for A ($A = \frac{1}{2}h^2$) if $h = 2\frac{1}{2}$?

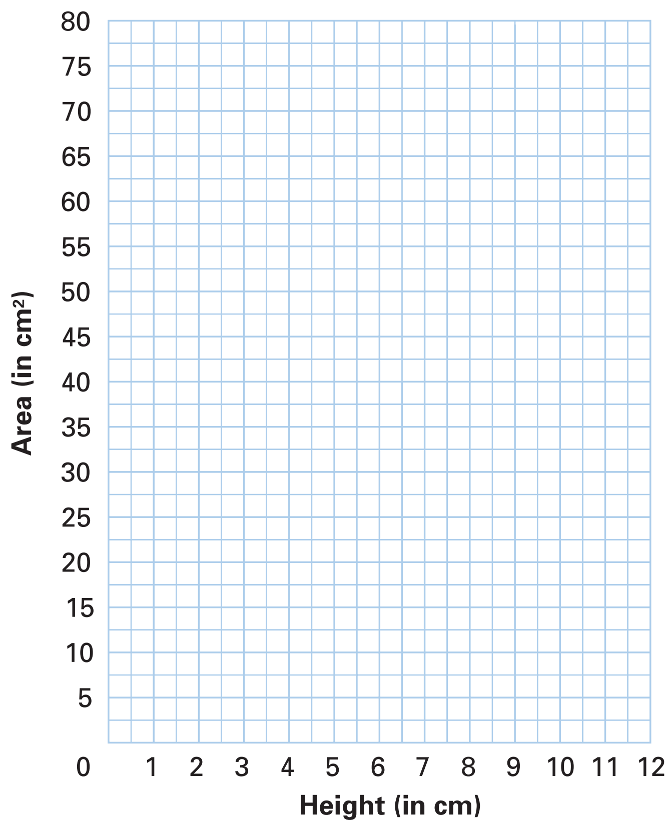
   b. How does the value of A for a poplar leaf change when you double the value of h? Use some specific examples to support your answer.

8. If the area of one poplar leaf is about 65 square centimeters (cm^2), what is its height? Explain how you found your answer.

The table shown below is also printed on **Student Activity Sheet 9**.

Height (in cm)	1	2	3	4	5	6	7	8
Area (in cm^2)	0.5	2	4.5					

Area of Black Poplar Leaves



9. **a.** Use **Student Activity Sheet 9** to fill in the remaining area values in the table. Use this formula:

$$A = \frac{1}{2}h^2$$

   b. Graph the formula on the grid. Why do you think the graph curves upward?

c. Using your graph, estimate the areas of poplar leaves with the following heights: 5.5 cm, 9.3 cm, and 11.7 cm.

d. Check your answers to part **c**, using the formula for the area of a poplar leaf. Which method do you prefer for finding the area of a poplar leaf given its height: the graph or the formula? Explain.

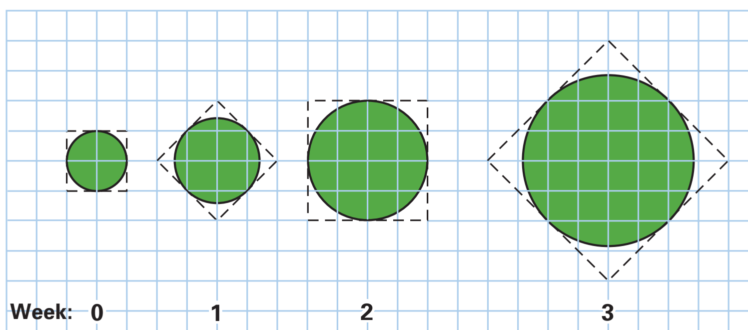
If the second differences in the table are equal, the growth is **quadratic**.

Water Lily

The *Victoria regina*, named after Queen Victoria of England, is a very large water lily that grows in South America. The name was later changed to *Victoria amazonica*. The leaf of this plant can grow to nearly 2 meters (m) in diameter.

10. How many of these full-grown leaves would fit on the floor of your classroom without overlapping?

Suppose you investigated the growth of a *Victoria amazonica* leaf and drew the following pictures on graph paper, one for every week.



11. How can you tell that the radius of the leaf does not grow linearly? Use the pictures shown above.

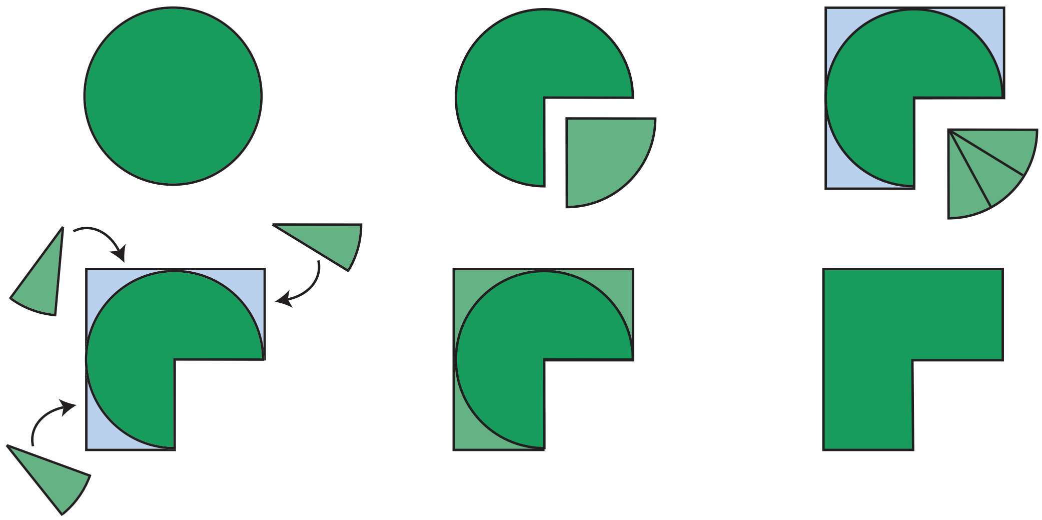
Use **Student Activity Sheet 10** for problem 12.

12. a. Make a table showing the length of the radius (in millimeters) of the lily each week. Try to make the numbers as accurate as possible.

b. Use the *first* and *second* differences in the table to find the next two entries in the table.

c. What can you conclude about the growth of the radius of the water lily?

The area of the leaves of *Victoria amazonica* is more important than the length of the radius if you want to compare the sizes of leaves while they grow. The following series of pictures shows one way of approximating the area of a circle if you know the radius.



Using these pictures, you can show that if a circle has a radius of five units, the area is about 75 square units.

13. **a.** Explain how the pictures show the area is about 75 square units.

b. Describe how you can find the area of a circle if you know its radius is ten units.

c. Describe how you can find the area of any circle if you know the radius.

To find the area of a circle, you can use the general rule you found in the previous problem. The formula below is more accurate:

area of a circle $= \pi \times r \times r$, or

area of a circle $\approx 3.14 \times r \times r$, where r is the radius of the circle

14. **a.** Use the answers you found in problem 12. Make a new table showing the area of the leaves of the water lily each week.

b. Does the area of the leaves show linear or quadratic growth? Explain your answer.

Aquatic Weeds

The waterweed *Salvinia auriculata,* found in Africa, is a fast-growing weed. In 1959, a patch of *Salvinia auriculata* was discovered in Lake Kariba on the border of what are now Zimbabwe and Zambia. People noticed it was growing very rapidly.

The first time it was measured, in 1959, it covered 199 square kilometers (km^2). A year later, it covered about 300 km^2. In 1963, the weed covered 1,002 km^2 of the lake.

15. **a.** Did the area covered by the weed grow linearly? Explain.

b. Was the growth quadratic? Explain.

Lake Victoria is about 1,000 miles north of Lake Kariba. Suppose a different weed were found there.

Here is a map of Lake Victoria with a grid pattern drawn on it. One square of this grid is colored in to represent the area covered by the weed in one year.

Use **Student Activity Sheet 11** to answer problems 16–19.

Suppose the area of the weeds in Lake Victoria doubles every year.

16. If the shaded square represents the area currently covered by the weed, how many squares would represent the area covered next year? A year later? A year after that?

Angela shows the growth of the area covered by the weed by coloring squares on the map. She uses a different color for each year. She remarks, "The number of squares I color for a certain year is exactly the same as the number already colored for all of the years before."

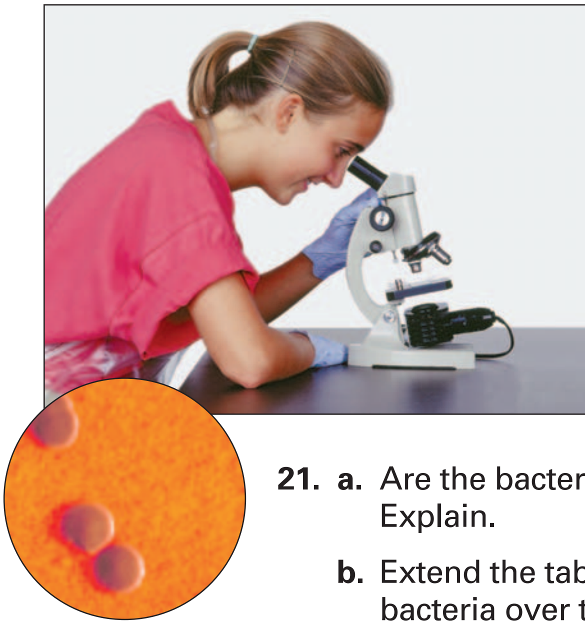
17. Use **Student Activity Sheet 11** to show why Angela is or is not correct.

18. How many years would it take for the lake to be about half covered?

19. How many years would it take for the lake to be totally covered?

Double Trouble

Carol is studying a type of bacteria at school. Bacteria usually reproduce by cell division. During cell division, a bacterium splits in half and forms two new bacteria. Each bacterium then splits again, and so on. These bacteria are said to have a **growth factor** of two because their amount doubles after each time period.



Suppose the number of bacteria doubles every 20 minutes.

20. Starting with a single bacterium, calculate the number of existing bacteria after two hours and 40 minutes. Make a table like the one below to show your answer.

Time (in minutes)	0	20	40
Number of Bacteria			

21. **a.** Are the bacteria growing linearly or in a quadratic pattern? Explain.

b. Extend the table to graph the differences in the number of bacteria over the course of two hours and 40 minutes.

c. Describe the graph.

22. **Reflect** How will the table and the graph in problem 21 change if the growth factor is four instead of two?

Two thousand bacteria are growing in the corner of the kitchen sink. You decide it is time to clean your house. You use a cleanser on the sink that is 99% effective in killing bacteria.

23. How many bacteria survive your cleaning?

24. If the number of bacteria doubles every 20 minutes, how long will it take before there are as many bacteria as before?

25. Find a NEXT-CURRENT formula for the growth of the bacteria.

This type of growth, where each new value is found by multiplying the previous number by a growth factor, is called **exponential growth**.

Time (in hours)	0	1	2	3
Number of Bacteria	1	6	36	216

$\times$ **6** $\times$ **6** $\times$ **6**

C Differences in Growth

Summary

Growth can be described by a table, a graph, or an equation. If you look at differences in a table, you know that:

- if the first differences are equal, the growth is linear. The rate of change is constant. In the table, the height of a plant is measured each week, and each week the same length is added to the previous length.

Time (in weeks)	0	1	2	3	4	5
Height (in cm)	10	15	20	25	30	35

+5 +5 +5 +5 +5

- if the first differences are not equal but the second differences are equal, the growth is quadratic.

In the table it shows the relationship between length and area of squares. If the length increases by 1 cm each time, the "second differences" are equal.

Length of Square (in cm)	1	2	3	4	5	6
Surface Area of Square (in cm^2)	1	4	9	16	25	36

+3 +5 +7 +9 +11

+2 +2 +2 +2

The table below shows that the area of a weed is being measured yearly and that the weed has a growth factor of two. Notice that having a growth factor of two means that the area covered by the weed is doubling every year.

Year	1	2	3	4	5
Area (in km^2)	400	800	1,600	3,200	6,400
	× 2	× 2	× 2	× 2	

- if each value in the table is found by multiplying by a growth factor, the growth is exponential.

Check Your Work

Consider the following formula for the area of a leaf: $A = \frac{1}{3}h^2$

Height (h) is measured in centimeters; area (A) is measured in square centimeters.

1. a. Use the formula for the area of a leaf to fill in the missing values in the table.

h (in cm)	3	4	5		7	8
A (in cm^2)				12		

b. Is the growth linear, quadratic, or exponential? Why?

The next table shows a model for the growth of dog nails.

Time (in months)	0	1	2	3	4	5
Length (in mm)	15	15.5	16	16.5	17	17.5

2. a. How can you be sure the growth represented in the table is linear?

b. What is the increase in length of the dog nail per month?

C Differences in Growth

Use this formula for the next problem:

area of a circle $\approx 3.14 \times r \times r$, where r is the radius of the circle

3. a. Find the area of a circle with the radius 2.5. You may use a calculator.

b. How many decimal places did you use for your answer? Explain why you used this number of decimal places.

c. Find the radius of a circle if the area is 100 cm^2. You may use a calculator. Show your calculations.

Suppose that you are offered a job for a six-month period and that you are allowed to choose how you will be paid:

$1,000 every week	or	1 cent the first week, 2 cents the second week, 4 cents the third week, 8 cents the fourth week, and so on…

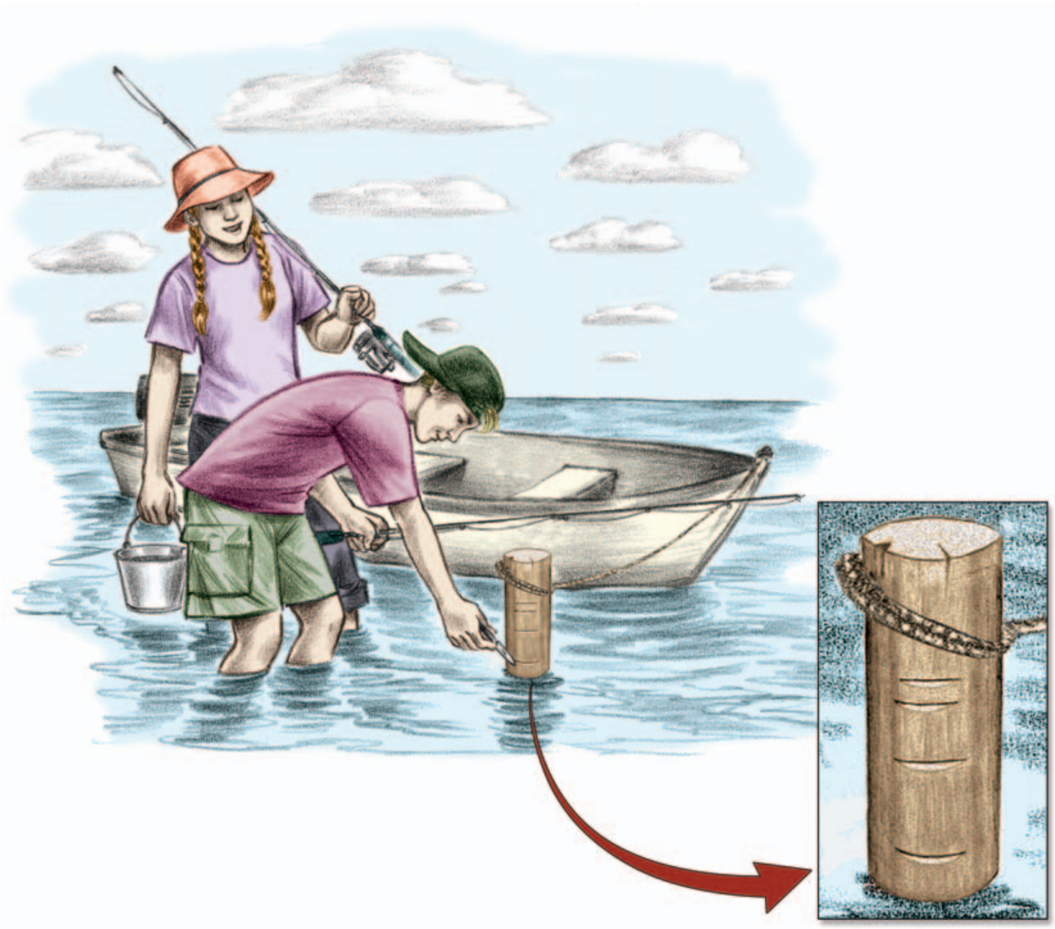
4. Which way of being paid would you choose? Why?

For Further Reflection

From birth to age 14, children grow taller. Think about your own growth during that time and describe whether you think it is linear, quadratic, exponential, or other. Give specific examples.

D Cycles

Fishing



Camilla and Lewis are fishing in the ocean. Their boat is tied to a post in the water. Lewis is bored because the fish are not biting, so he decides to amuse himself by keeping track of changes in the water level due to the tide.

He makes a mark on the post every 15 minutes. He made the first mark (at the top) at 9:00 A.M.

1. What do the marks tell you about the way the water level is changing?

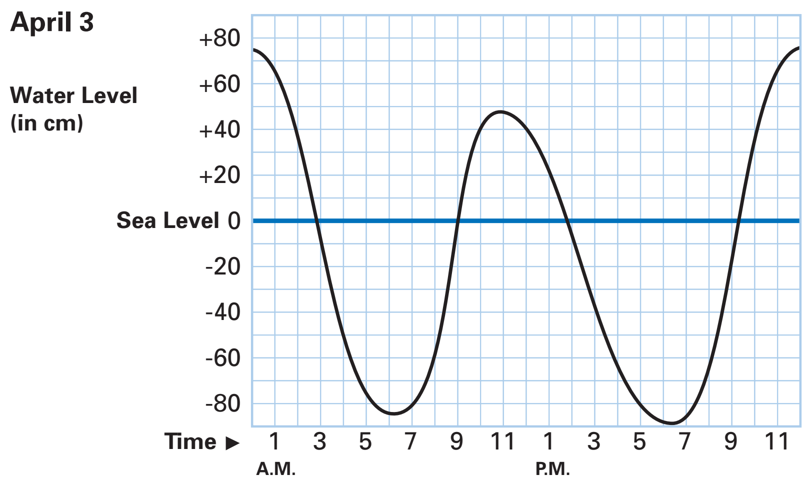
2. Make a graph that shows how the water level changed during this time.

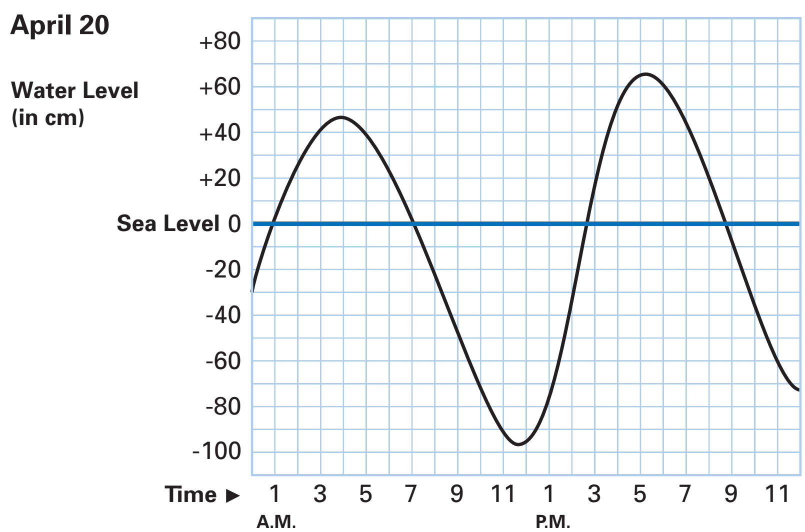
3. What can you say about how the graph will continue? (Think about the tides of the ocean.) You don't need to graph it.

High Tide, Low Tide

During low tide naturalists lead hikes along coastal tide flats. They point out the various plant and animal species that live in this unique environment. Some of the plants and animals hikers see on the walk are seaweed, oystercatchers, curlews, plovers, mussels, jellyfish, and seals.

Walkers do not always stay dry during the walk; sometimes the water may even be waist-high. Walkers carry dry clothing in their backpacks and wear tennis shoes to protect their feet from shells and sharp stones.





The graphs show the tides for the tide flats for two days in April. Walking guides recommend that no one walk in water deeper than 45 cm above the lowest level of the tide.

4. When is it safe to walk on the tide flats on these two days in April?

Golden Gate Bridge

In different parts of the world, the levels of high and low tides vary. The amount of time between the tides may also vary. Here is a tide schedule for the area near the Golden Gate Bridge in San Francisco, California.

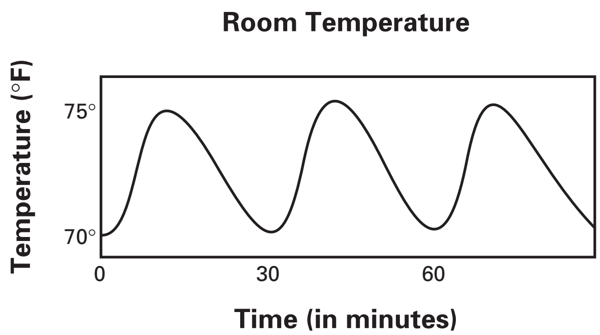
5. Use the information in the table to sketch a graph of the water levels near the Golden Gate Bridge for these three days. Use **Student Activity Sheet 12** for your graph.

Date	Low	High
Aug. 7	2:00 A.M./12 cm 1:24 P.M./94 cm	9:20 A.M./131 cm 7:47 P.M./189 cm
Aug. 8	2:59 A.M./6 cm 2:33 P.M./94 cm	10:18 A.M./137 cm 8:42 P.M./189 cm
Aug. 9	3:49 A.M./0 cm 3:29 P.M./91 cm	11:04 A.M./143 cm 9:34 P.M./186 cm

6. Describe how the water level changed.

7. Compare your graph to the graphs on the previous page. What similarities and differences do you notice?

The Air Conditioner



Suppose the graph on the left shows the temperature changes in an air-conditioned room.

8. Describe what is happening in the graph. Why do you think this is happening?

The graphs you have seen in this section have one thing in common: They have a shape that repeats. A repeating graph is called a **periodic graph**. The amount of time it takes for a periodic graph to repeat is called a **period** of the graph. The portion of the graph that repeats is called a **cycle**.

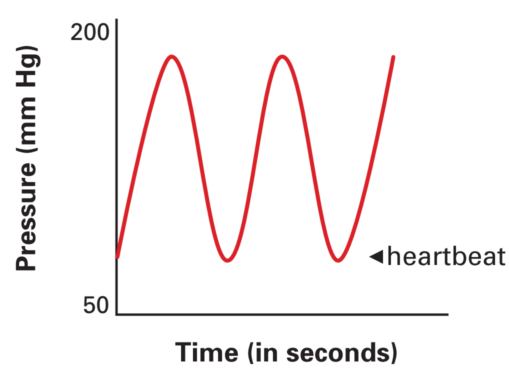
9. a. How long (in minutes) is a period in the above graph?

b. On **Student Activity Sheet 13**, color one cycle on the graph.

Blood Pressure

Your heart pumps blood throughout your system of arteries. When doctors measure blood pressure, they usually measure the pressure of the blood in the artery of the upper arm.

Your blood pressure is not constant. The graph on the right shows how blood pressure may change over time.



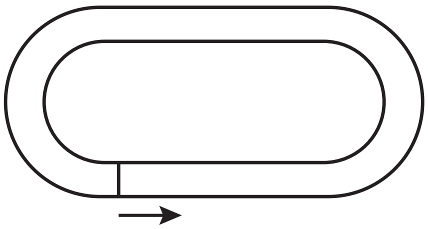
10. What can you tell about blood pressure just before a heartbeat?

11. What happens to blood pressure after a heartbeat?

12. Is this graph periodic? Explain your answer.

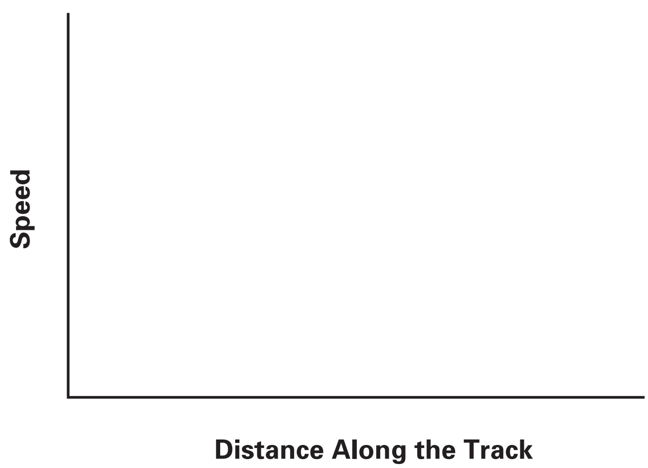
The Racetrack

A group of racecar drivers are at a track getting ready to practice for a big race. The drawing shows the racetrack as seen from above.



13. Make a sketch of the track in your notebook. Color it to show where the drivers should speed up and where they should slow down. Include a key to show the meanings of the colors or patterns you chose.

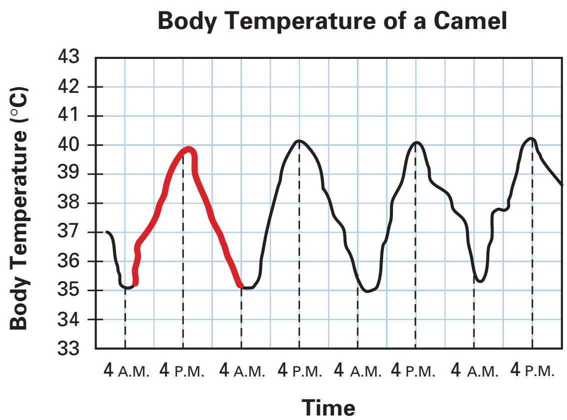
14. Make a graph of the speed of a car during three laps around the track. Label your graph like the one here.



15. Explain why your graph is or is not periodic.

D Cycles

Summary

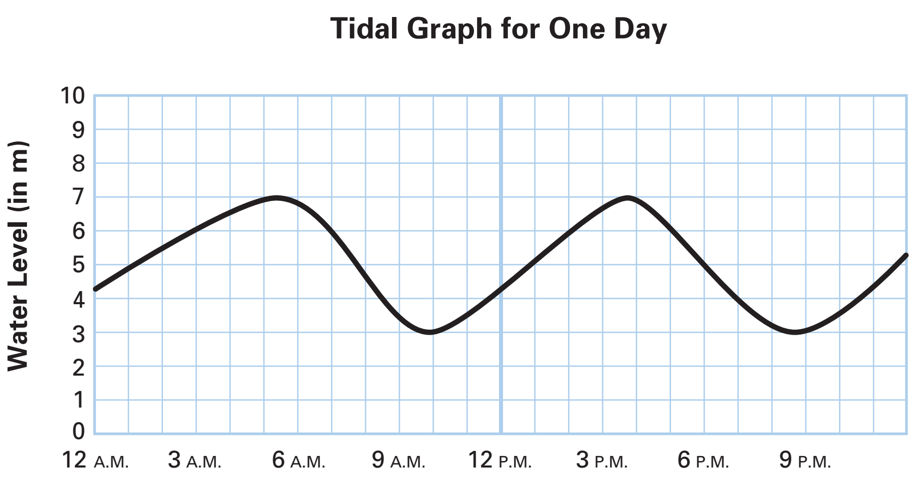


A *periodic graph* shows a repeating pattern. In real life, there can be small changes in the pattern.

A *period* on the graph is the length of time or the distance required to complete one *cycle,* or the part of the pattern that is repeated. One cycle is indicated on the graph; the period is 24 hours, or one day.

Check Your Work

The graph shows the height of the seawater near the port of Hoeck in The Netherlands.



1. a. Is this graph a periodic graph? Why?

   b. How many hours after the start of this graph was low tide?

   c. What is the depth of the water during low tide?

   d. How many hours pass between two high tides?

   e. What is the period?

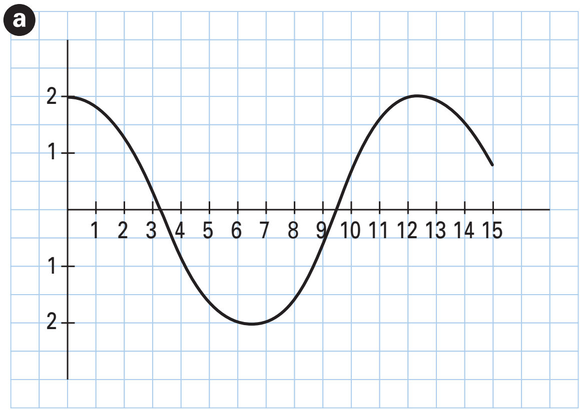
In a refrigerator, the temperature is not always the same. Even if the door is closed all the time, the temperature will slowly rise. In some refrigerators, as soon as the temperature reaches 45°F, the refrigeration system starts to work. The temperature will decrease until it reaches 35°F. In general, this takes about 10 minutes. Then the temperature starts rising again until it reaches 45°F. This takes about 20 minutes. Then the whole cycle starts all over again.

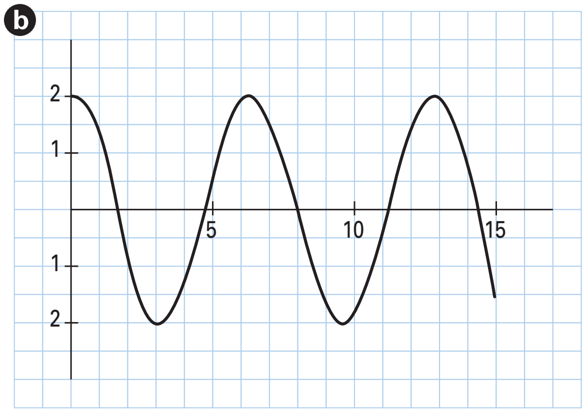
2. **a.** Draw a graph that fits the above information about the refrigeration system. Make your graph as accurately as possible. Be sure to label the axes.

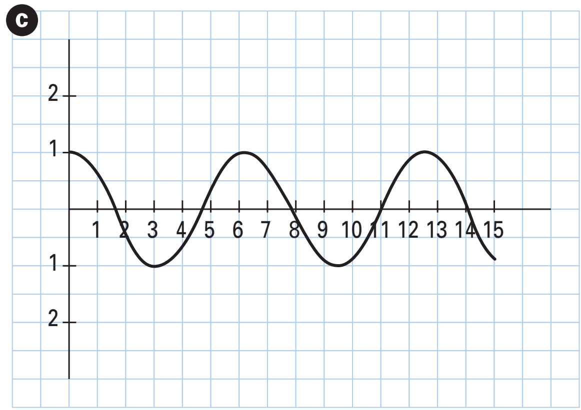
b. How many minutes does one complete cycle take?

Three periodic graphs are shown.

3. Which graph shows a situation that has a period of about six? Explain your reasoning.

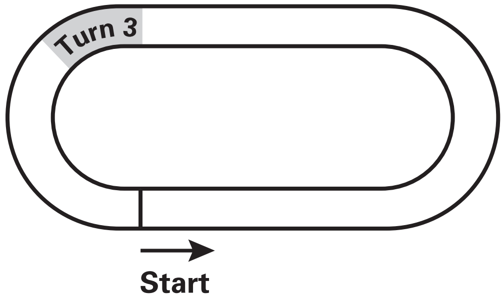


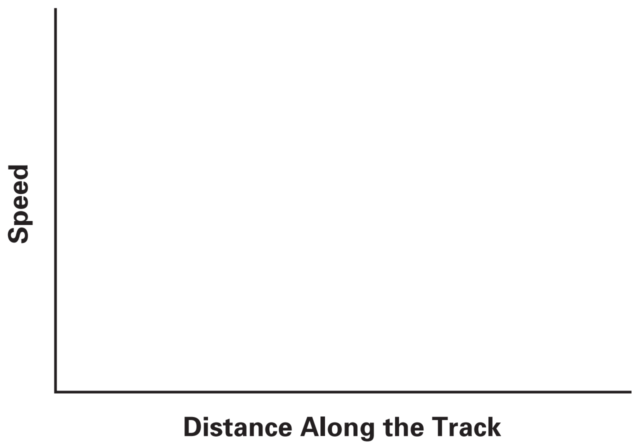




For Further Reflection

Refer to the graph of the speed of a racecar. Describe the appearance of the graph if the racecar ran out of gas at turn 3.



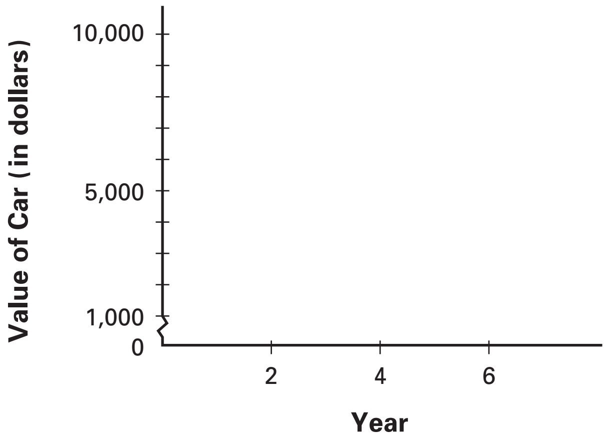


E Half and Half Again

Fifty Percent Off

Monica is shopping for a used car. She compares the prices and ages of midsize cars. She notices that adding two years to the age of a car lowers the price by 50%.

1. Copy the diagram below. Graph the value of a $10,000 car over a six-year period.
2. Is the graph linear? Why or why not?



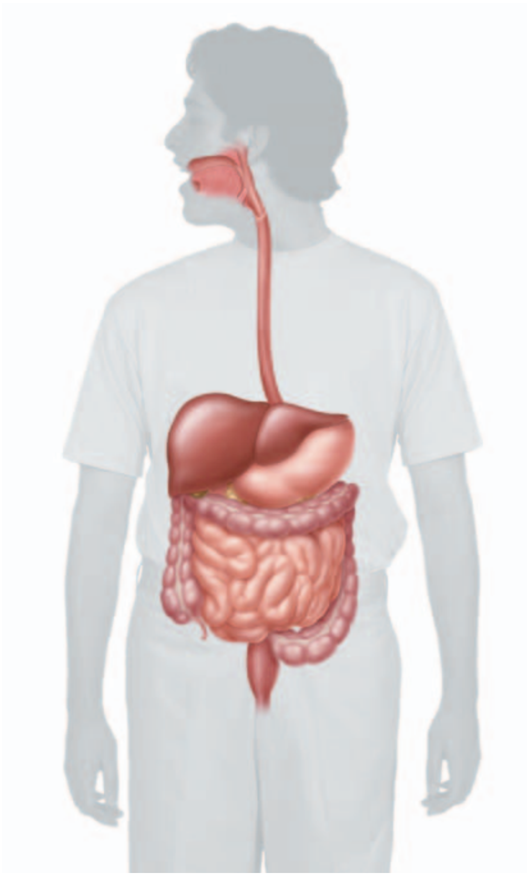
Monica decides she does not want to keep a car for more than two years. She needs advice on whether to buy a new or used car.

3. Write a few sentences explaining what you would recommend. Support your recommendation.

4. a. **Reflect** Show that at this rate, the car never has a zero value.

   b. Is this realistic?

Medicine



When you take a certain kind of medicine, it first goes to your stomach and then is gradually absorbed into your bloodstream. Suppose that in the first 10 minutes after it reaches your stomach, half of the medicine is absorbed into your bloodstream. In the second 10 minutes, half of the remaining medicine is absorbed, and so on.

5. What part of the medicine is still in your stomach after 30 minutes? After 40 minutes? You may use drawings to explain your answer.

6. What part of the medicine is left in your stomach after one hour?

Kendria took a total of 650 milligrams (mg) of this medicine.

7. Copy the table. Fill in the amount of medicine that is still in Kendria's stomach after each ten-minute interval during one hour.

Minutes after Taking Medicine	0	10	20	30	40	50	60
Medicine in Kendria's Stomach (in mg)	650						

8. Graph the information in the table you just completed. Describe the shape of the graph.

The time it takes for something to reduce by half is called its half-life.

9. Is the amount of medicine in Kendria's stomach consistent with your answer to problem 6? Explain.

Suppose that Carlos takes 840 mg of another type of medicine. For this medicine, half the amount in his stomach is absorbed into his bloodstream every two hours.

10. Copy and fill in the table to show the amounts of medicine in Carlos's stomach.

Hours after Taking Medicine	0	2	4	6	8	10	12
Medicine in Carlos's Stomach (in mg)	840						

11. **a.** How are the succeeding entries in the table related to one another?

b. Find a NEXT-CURRENT formula for the amount of medicine in Carlos's stomach.

c. Does the graph show linear growth? Quadratic growth? Explain.

The table in problem 10 shows **negative growth**.

12. **a.** **Reflect** Explain what negative growth means.

b. What is the growth factor?

c. Do you think the growth factor can be a negative number? Why or why not?

In Section C, you studied examples of exponential growth with whole number growth factors. The example above shows exponential growth with a positive growth factor less than one. This is called **exponential decay**.

E Half-Lives

Summary

You have explored several situations in which amounts have decreased by a factor of $\frac{1}{2}$. For example, the price of a car decreased by a factor of $\frac{1}{2}$ every two years. The amount of medicine in the stomach decreased by a factor of $\frac{1}{2}$ every 10 minutes, or every two hours.

Sometimes things get smaller by half and half again and half again and so on. When things change this way, the change is called *exponential decay*. When amounts decrease by a certain factor, the growth is not linear and not quadratic. You can check by finding the *first* and *second differences*.

Check Your Work

Suppose you have two substances, A and B. The amount of each substance changes in the following ways over the same length of time:

Substance A: NEXT = CURRENT × $\frac{1}{2}$

Substance B: NEXT = CURRENT × $\frac{1}{3}$

1. Are the amounts increasing or decreasing over time?

2. Which of the amounts is changing more rapidly, A or B? Support your answer with a table or a graph.

Although one of the amounts is changing more rapidly, both are decreasing in a similar way.

3. a. How would you describe the way they are changing—faster and faster, linearly, or slower and slower? Explain.

   b. **Reflect** What is the mathematical name for this type of change?

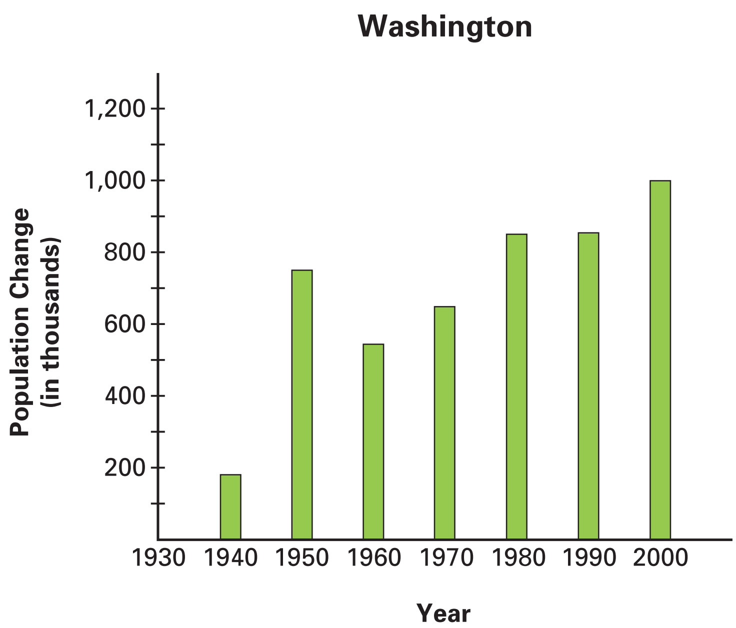
For Further Reflection

Write a description of exponential decay using pesticides.

Additional Practice

Section A Trendy Graphs

The graph on the left shows how much the population of the state of Washington grew each decade from 1930 to 2000.



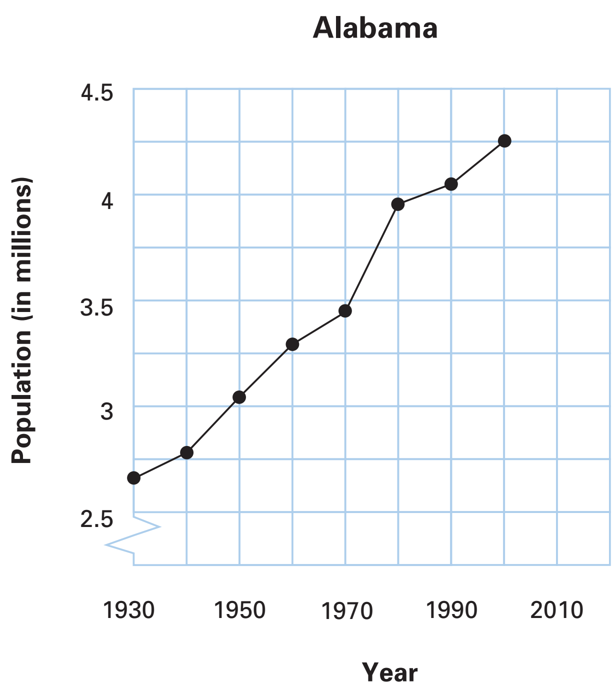
1. a. By approximately how much did the population grow from 1930 to 1940?

   b. In 1930, the population of Washington was 1,563,396. What was the approximate population of Washington in 1940?

2. a. During which decade did the population grow the most? Explain.

   b. When did the population grow the least? Explain.

The graph below shows the growth of Alabama's population from 1930 to 2000.



3. a. Describe the growth of the population from the year 1960 until the year 2000.

   b. From 1970 to 1980, the population of Alabama grew fast. How does the graph show this?

Additional Practice

These are two pictures of the same iguana.

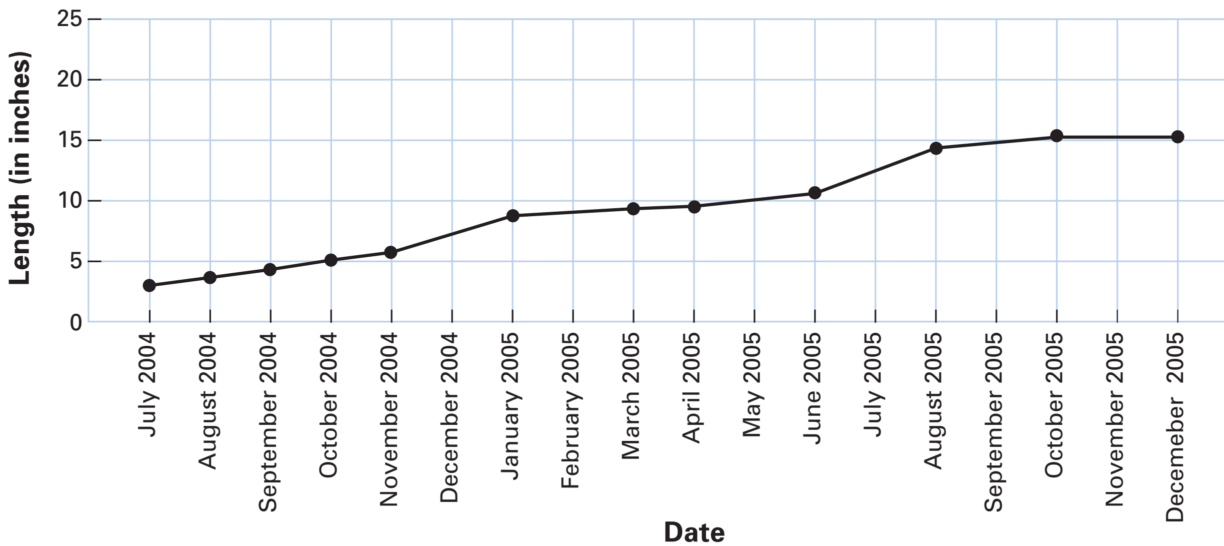
The chart shows the length of the iguana as it grew.

Date	Length (in inches)	
	Overall	Body (without tail)
July 2004	$11\frac{1}{2}$	3
August 2004	13	$3\frac{1}{2}$
September 2004	15	4
October 2004	17	5
November 2004	$21\frac{1}{2}$	$5\frac{1}{2}$
January 2005	27	$7\frac{1}{2}$
March 2005	$29\frac{1}{2}$	$8\frac{1}{2}$
April 2005	31	9
June 2005	$38\frac{1}{2}$	$11\frac{1}{2}$
August 2005	45	14
October 2005	$49\frac{1}{2}$	$15\frac{1}{2}$
December 2005	50	$15\frac{1}{2}$

Note that the iguana was not measured every single month.

The graph of the length of the iguana's body, without the tail, is drawn below.

Growth of an Iguana



4. **a.** Use graph paper to draw the line graph of the overall length of the iguana. Be as accurate as possible.

   b. Use your graph to estimate the overall birth length of the iguana in June 2004.

   c. On November 1, 2005, the iguana lost part of its tail. Use a different color to show what the graph of the overall length may have looked like between October and December 2005.

Section B Linear Patterns

Mark notices that the height of the water in his swimming pool is low; it is only 80 cm high. He starts to fill up the pool with a hose. One hour later, the water is 95 cm high.

1. If Mark continues to fill his pool at the same rate, how deep will the water be in one more hour?

2. **a.** If you know the current height of the water, how can you find what the height will be in one hour?

   b. Write a NEXT-CURRENT formula for the height of the water.

3. Mark wants to fill his pool to 180 cm. How much time will this take? Explain.

Mark has already spent three hours filling up his pool. He wants to fill up the pool faster, so he uses another hose. With the two hoses, the water level rises 25 cm every hour.

4. On graph paper, draw a graph showing the height of the water in Mark's pool after he starts using two hoses.

The following formula gives the height of the water in Mark's pool after he starts using two hoses.

$$H = \_\_\_\_\_ + 25T$$

5. a. What do the letters H and T represent?

b. A number is missing in the formula. Rewrite the formula and fill in the missing number.

c. If Mark had not used a second hose, how would the formula in part **b** be different?

Section C Differences in Growth

Food in a restaurant must be carefully prepared to prevent the growth of harmful bacteria. Food inspectors analyze the food to check its safety. Suppose that federal standards require restaurant food to contain fewer than 100,000 salmonella bacteria per gram and that, at room temperature, salmonella has a growth factor of two per hour.

1. There are currently 200 salmonella bacteria in 1 g of a salad. In how many hours will the number of bacteria be over the limit if the salad is left at room temperature?

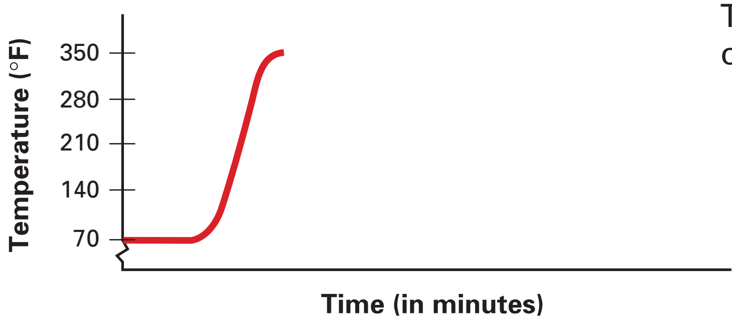
A food inspector found 40,000,000 salmonella bacteria in 1 g of chocolate mousse that had been left out of the refrigerator.

2. Assume that the mousse had been left at room temperature the entire time. What level of bacteria would the food inspector have found for the chocolate mousse one hour earlier? One hour later?

3. Write a NEXT-CURRENT formula for the number of salmonella bacteria in a gram of food kept at room temperature.

4. When the chocolate mousse was removed from the refrigerator, it had a safe level of salmonella bacteria. How many hours before it was inspected could it have been removed from the refrigerator?

Section D Cycles



The graph shows the temperature of an oven over a period of time.

1. Describe what the red part of the graph tells you about the temperature of the oven.

2. Copy the graph and show how the temperature changes in the oven as the heating element shuts on and off.
3. Show what happens when someone turns the oven off.
4. Color one cycle on your graph.

Section E Half and Half Again

Boiling water (water at 100°C) cools down at a rate determined by the air temperature. Suppose the temperature of the water decreases by a factor of $\frac{1}{10}$ every minute if the air temperature is 0°C.

1. Under the above conditions, what is the temperature of boiling water one minute after it has started to cool down? Two minutes after? Three? Four?

Examine the following NEXT-CURRENT formulas.

2. Which ones give the temperature for water that is cooling down if the air temperature is 0°C? Explain.

   a. NEXT = CURRENT − 10

   b. NEXT = CURRENT × 0.9

   c. NEXT = CURRENT − CURRENT × 0.1

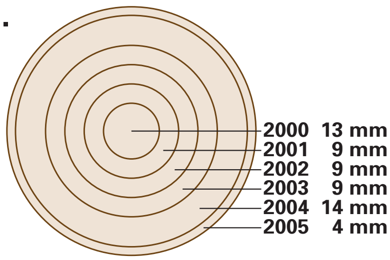
   d. NEXT = CURRENT × 0.1

3. How long does it take for boiling water to cool down to 40°C if the air temperature is 0°C?

Answers to Check Your Work

Section A Trendy Graphs

1. a.

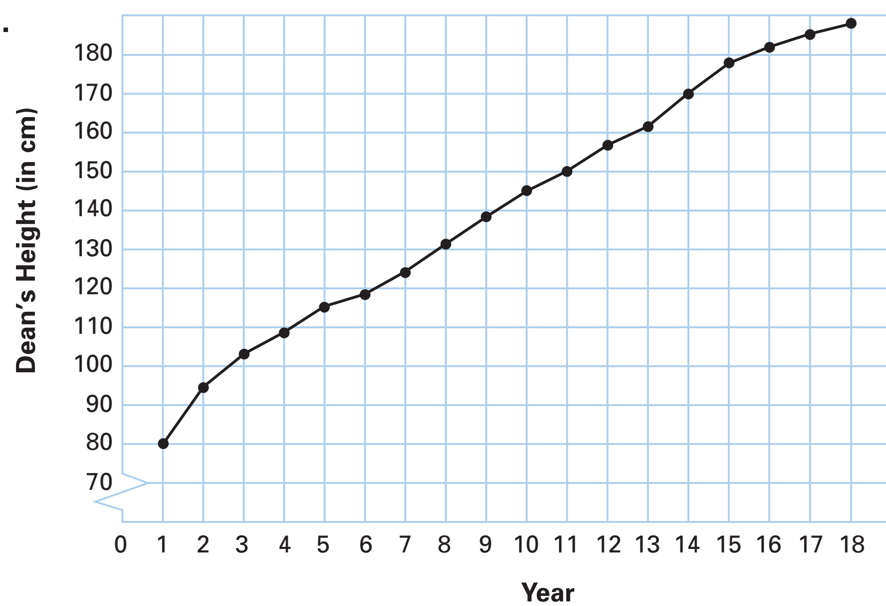


b. Your story may differ from the samples below.

First it was planted. It got plenty of sun, air, and water, and it grew a lot. It grew the second year, but not as much as the first year. The third and fourth years, the tree grew about the same as the second year. The fifth year, the tree grew about the same as the first year. The sixth year, the tree either had a disease or did not get enough sun or water.

I planted this tree in 2000. For the first year, I watered it a lot and took care of it. It was a very pretty tree and grew a lot in the first year. The second, third, and fourth years, I got really bored with it and stopped watering it. It didn't grow much those years. Maybe it grew a couple of inches, but that's all. In 2004, I decided that my tree was very special, and I started to water it more. It really grew that year. But the next year, I got too busy to water it very much, and it grew very little.

2. a.



b. Discuss your answer with a classmate. Sample answers:

- From Dean's second birthday until his sixteenth birthday, he grew regularly.
- After his sixteenth birthday, Dean's growth started to slow down, but he may still get taller after his nineteenth birthday.

c. According to the graph, Dean had his biggest growth spurt between his first and second birthdays. Dean grew 15 cm that year. But his length at birth is missing from the graph, so maybe he had his biggest growth spurt during his first year.

3. a. The first tree will have the larger circumference. You may give an explanation by looking at the table, or you may make a graph and reason about the trend this graph shows.
Sample responses:

- Looking for patterns in the table:
The first tree is growing by an increasing amount every year, while the second tree is growing by a decreasing amount every year. You can see this in the tables using arrows with numbers that represent the differences.

	Circumference (in inches)		
	First Measurement	Second Measurement	Third Measurement
Tree 1	2.0	3.0	4.9

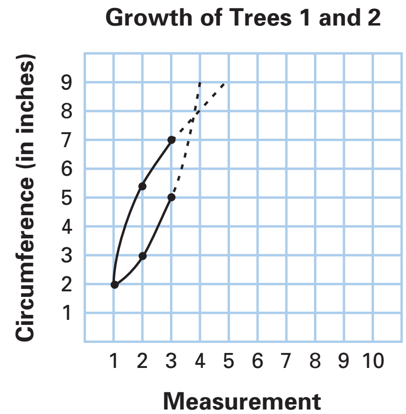
+1.0 in +1.9 in

	Circumference (in inches)		
	First Measurement	Second Measurement	Third Measurement
Tree 2	2.0	5.5	7.1

+3.5 in +1.6 in

- The first tree might grow as much as 4 in., putting it at 8.9 in. The second tree will probably grow less than 1 in, putting it at about 8 in. Make two graphs and reason about the trend the graphs show.

b. You may prefer either the graph or the table. You might prefer the table because it has the actual numbers, and you can calculate the exact change each year and use those numbers to make your decision. You might prefer the graph because you can see the trend in the growth of each tree and also the relationship between the two trees.



Section B Linear Patterns

1. a. table:

Time (in weeks)	0	1	2	3	4	5	6
Earnings (in dollars)	0	12	24	36	48	60	72

It is all right if you started your table with week 1.

b. recursive formula: NEXT = CURRENT + 12

c. direct formula: $E = 12W$, with E in dollars and W in weeks

2. a. A table will show that the length grows each month by the same amount; the differences are all equal to 1.4 cm.

b. NEXT = CURRENT + 1.4

c. $L = 11 + 1.4T$, with L in centimeters and T in months

3. The first formula gives Sonya's hair growth each year, so *NEXT* stands for next year, *CURRENT* stands for the current year, and 14.4 stands for the number of centimeters her hair grows yearly.

The second formula gives her hair growth each month, so *NEXT* stands for next month, *CURRENT* stands for the current month, and 1.2 stands for the number of centimeters her hair grows monthly.

4. Discuss your formula with a classmate. Your formula may differ from the examples shown below; you may have chosen other letters or used words. Sample formulas:

- $E = 12 + 10T$ with earnings E in dollars and time T in half hours
- $E = 12 + 20T$ with earnings E in dollars and time T in half hours
- *amount* (in dollars) = 12 + 10 (*time* (30 minutes)

5. a. Show your sign to your classmates.

b. Discuss your formula with a classmate or in class. There are different ways to make a formula that is yields a less costly result than Lamar's. You will have to show why it is less costly and give your reasoning. Some examples:

- Make both the charge for a house call and the amount per half hour lower than in Lamar's formula. You will always be cheaper. For instance: $E = 10 + 8T$ with earnings E in dollars and time T in half hours.
- Keep the call charge equal, but make the amount per half hour lower than in Lamar's formula. You will always be cheaper.
For instance: $E = 12 + 8T$ with earnings E in dollars and time T in half hours.
- Make the call charge higher and the amount per half hour lower than in Lamar's formula. For instance: $E = 20 + 8T$ with earnings E in dollars and time T in half hours.

  Your company will be cheaper after more than four half hours, but you will earn more for short calls. Make a note on your website that most jobs take an average of two hours.

Section C Differences in Growth

1. a. Remember that squaring a number goes before multiplying. An example, for $h = 4$:

$A = \frac{1}{3} \times (4^2) = \frac{1}{3} \times 4 \times 4 = \frac{16}{3}$

$A = 5\frac{1}{3}$ (Note that you should always write fractions in simplest form and change improper fractions to mixed numbers.)

h **(in cm)**	3	4	5	6	7	8
A **(in cm²)**	3	$5\frac{1}{3}$	$8\frac{1}{3}$	12	$16\frac{1}{3}$	$21\frac{1}{3}$

b. The first differences are: $2\frac{1}{3}$, 3, $3\frac{2}{3}$, $4\frac{1}{3}$, and 5; the growth is not linear because the first differences in the table are not equal.

The second differences are: $\frac{2}{3}$, $\frac{2}{3}$, $\frac{2}{3}$, and $\frac{2}{3}$; the growth is quadratic because the second differences are equal.

The growth is not exponential because the numbers in the second row are not multiplied by the same number to get from one to the next.

2. a. The first differences in the table are equal; they are 0.5. Therefore, you know the growth is linear.

b. The increase in length of the toenail each month is 0.5 mm.

3. a. $3.14 \times 2.5 \times 2.5 \approx 19.6$

Area of the circle is about 19.6.

Note that in the given radius of 2.5, no units were mentioned.

b. You may have noted that the answer 19.625 was shown in the calculator window.

However, because the radius is given in one decimal, the answer should also be in one decimal.

c. If you do not have a calculator, try some carefully chosen examples.

You know that $3 \times 25 = 75$, so $r > 5$.

$3 \times 36 = 108$, so $r < 6$; you now know that r is between 5 and 6.

Try $r = 5.5$.

$3.14 \times 5.5 \times 5.5 \approx 95$ (too little)

Try $r = 5.6$.

$3.14 \times 5.6 \times 5.6 \approx 98$ (too little)

Try $r = 5.7$.

$3.14 \times 5.7 \times 5.7 \approx 102$ (too much)

The answer will be $r \approx 5.6$ or $r \approx 5.7$.

Using a calculator:

$3.14 \times r \times r = 100$

$r \times r = 100 \div 3.14 = 31.847....$ (Don't round off until you have the final answer.)

Find a number that gives 31.847…. as a result if squared. Or: The square root (√) of 31.847…. is about 5.6.

Radius r is about 5.6.

4. Discuss your answer with a classmate. Sample calculations:

Doubling a penny adds up to more money in a six-month period than receiving $1,000 a week. I calculated how much money I would make after six months, using the first case: 26 weeks × $1,000 per week = $26,000.

For the second case, I calculated the amount I would receive week by week with a calculator:

In week ten, the amount would be $5.12, and all together I would have been paid $10.23.

In week twenty, the amount I would make would already be $5,242.88, and the total I would have earned would be $10,485.75.

After 23 weeks, my pay for one week would already be $41,943.04, so for that one week I would make more than I would in six months in the first case. So I would choose the doubling method.

Remembering that in the second way I have to find how much I would make each week and add up what I was already paid, in week 22, I would have made $20,971.52. My total earnings after that week would be $41,943.03. So the doubling method is better.

Section D Cycles

1. **a.** The graph is periodic; the same pattern is repeated.

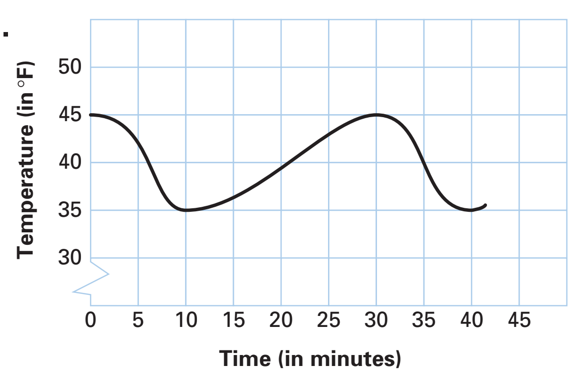
b. After about ten hours (or a little less than ten hours).

c. During low tide, the depth of the water is 3 m.

d. Ten hours pass between two high tides.

e. The period is one full cycle, so ten hours.

2. **a.**



b. One complete cycle takes 30 minutes.

3. Graphs **b** and **c** have a period of about six. Note that the period of graph **a** is about 12.5.

Section E Half and Half Again

1. The amounts are decreasing. Sample explanations:

   - For substance A, after every time interval, half of what there was is left; and for substance B, after every time interval, one-third of what there was is left.
   - You can see the decrease when you use the formulas to make tables.

Time	0	1	2
A	30	15	7.5

$\times\frac{1}{2}$ $\times\frac{1}{2}$

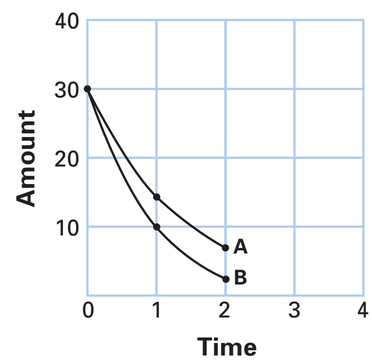
Time	0	1	2
B	30	10	3.3

$\times\frac{1}{3}$ $\times\frac{1}{3}$

2. The amount of substance B is decreasing more rapidly. You can see the decrease by looking at the numbers if you make a table for the answer to problem 1. The amount of B goes down faster.

   You can see that substance B is decreasing faster by looking at the two graphs.

Sample graph:



3. **a.** More and more slowly. Sample explanation:

   You can see that the decrease is happening more and more slowly by looking at the differences in the tables; for example:

Time	0	1	2
A	30	15	7.5

-15 $-7\frac{1}{2}$

Time	0	1	2
B	30	10	3.3

-20 -7

   If you look at the graphs, you can see that the decrease slows down and the graphs become "flatter" over time.

   b. This type of change is called exponential decay.